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Mathematical modelling of natural gas markets

BTU Cottbus & Gas Union
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Contents:

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Introduction to MCP formulations

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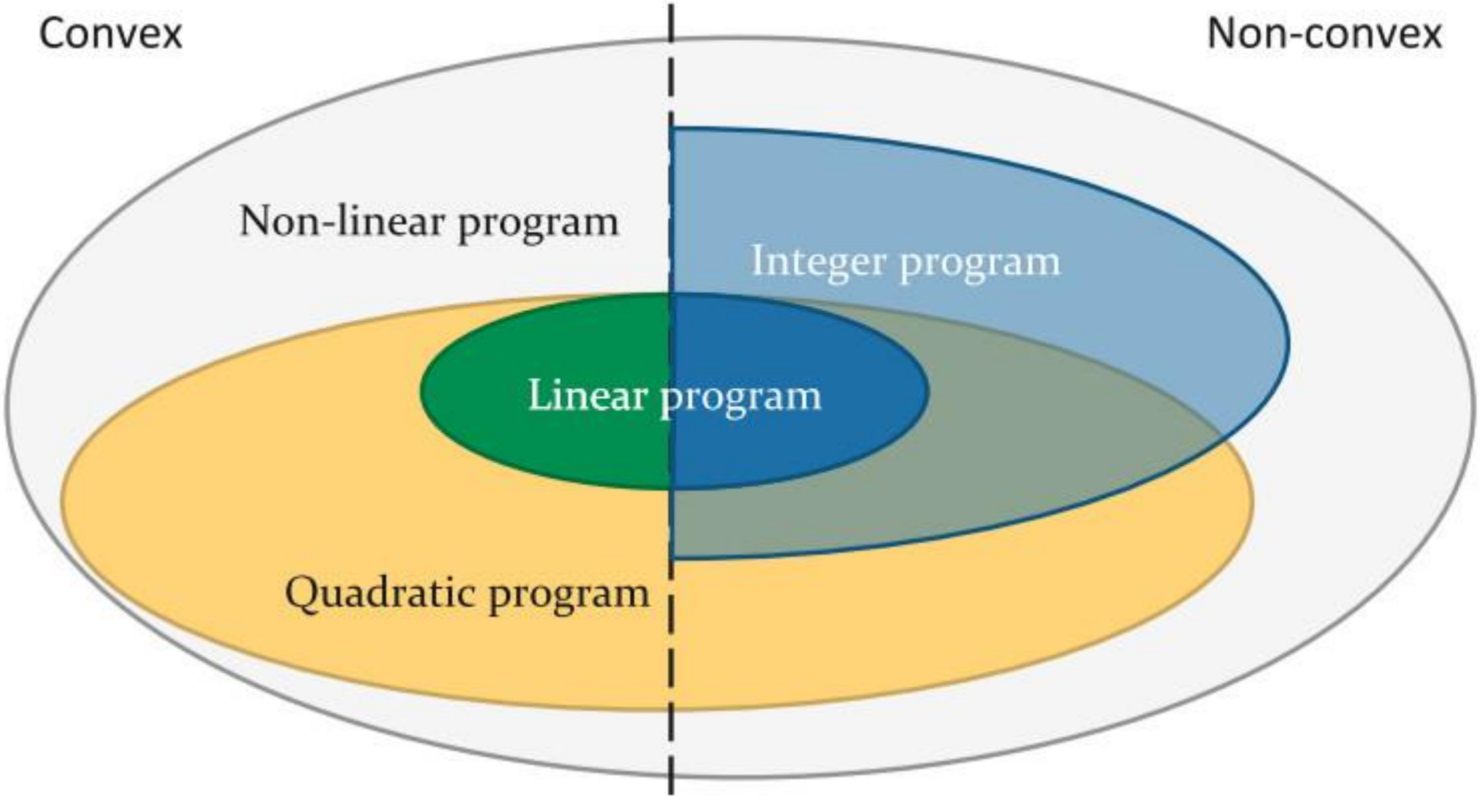
- MCP (mixed complementarity programming) is a common modelling approach to describe various energy markets around the world.
- Complementarity models generalize linear programs (LP), quadratic programs (QP) and (convex) nonlinear programs (NLPs)

Introduction to MCP formulations: types of optimization problems



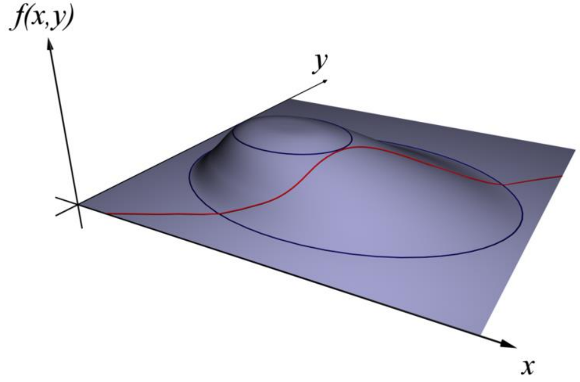
Source: [4]

Introduction to MCP formulations

- MCP (mixed complementarity programming) is a common modelling approach to describe various energy markets around the world.
- Complementarity models generalize linear programs (LP), quadratic programs (QP) and (convex) nonlinear programs (NLPs)
- Complementarity problems are appropriate for modelling the regulated/deregulated, perfect/imperfect competition that characterizes today's energy markets

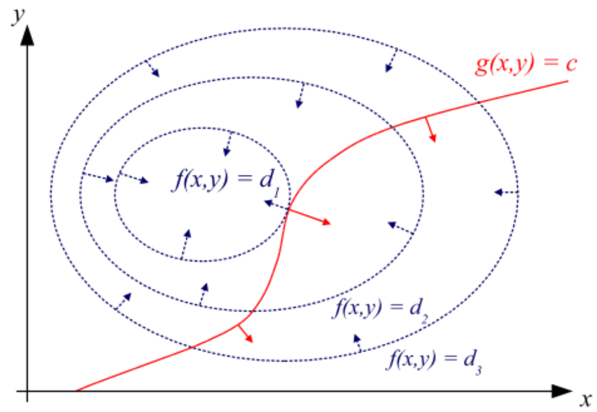
Method of Lagrange multipliers: problem definition

In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*



$$max\, f(x,y)$$

$$s.t.\, g(x,y) = c$$



Where f(x,y) – objective function
g(x,y) - constraint

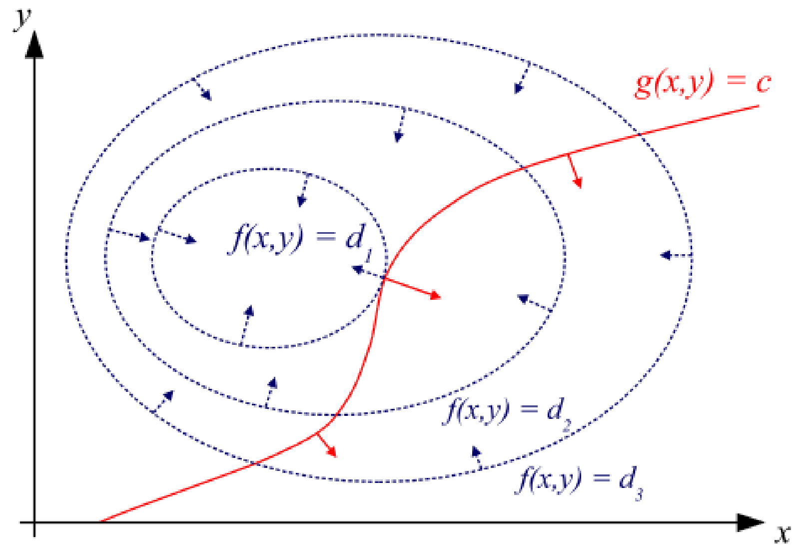
Method of Lagrange multipliers

In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*

$$max\, f(x, y)$$

$$s.t.\, g(x, y) = c$$

Key point: 2 curves are tangent at the same point -> i.e. they have the same slope



Method of Lagrange multipliers

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In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*

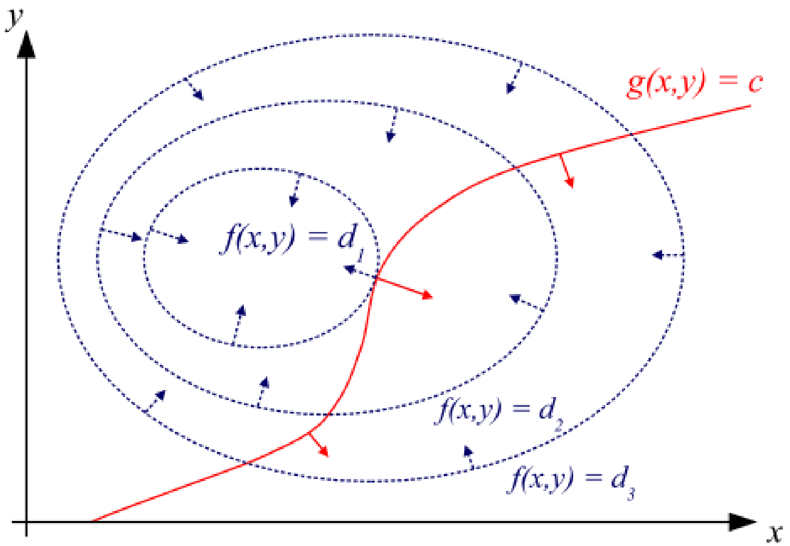
$$max\ f(x,y)$$

$$s.t.\ g(x,y) = c$$

$$\nabla f(x,y) = \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$$

$$\nabla g(x,y) = \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}$$

Gradient of the function shows the direction of the max increase of the function:



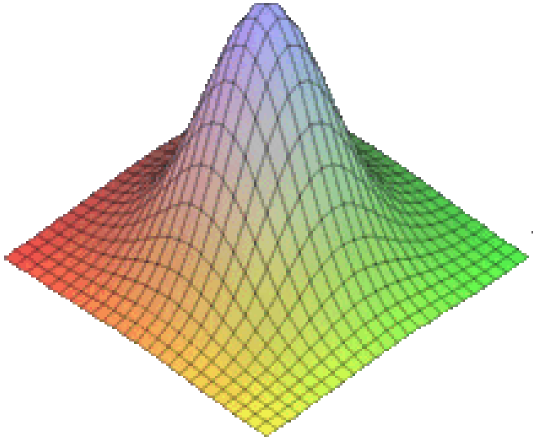
Method of Lagrange multipliers: Gradient

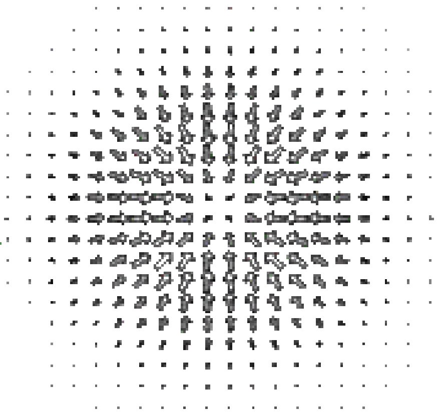
In mathematics, the gradient is a *generalization of the usual concept of derivative of a function in one dimension to a function in several dimensions.*

✓ Gradient points in the direction of the greatest rate of increase of the function and its magnitude is the slope of the graph in that direction

$$\nabla f = \frac{\partial f}{\partial x_1}\mathbf{e}_1 + \cdots + \frac{\partial f}{\partial x_n}\mathbf{e}_n$$

where the e_i are the orthogonal unit vectors pointing in the coordinate directions.





Method of Lagrange multipliers

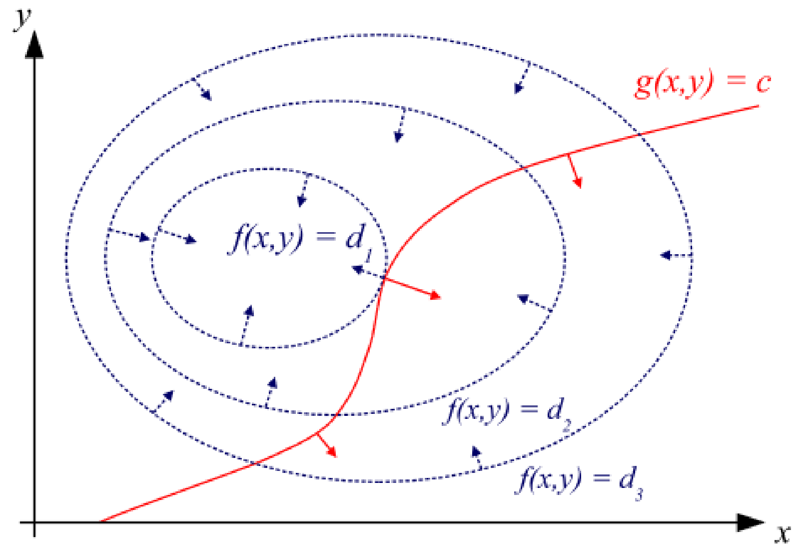
In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*

$$max\, f(x,y)$$

$$s.t.\, g(x,y) = c$$

$$\nabla f(x,y) = \lambda \cdot \nabla g(x,y)$$

If 2 vectors are orthogonal to the same slope, it has to be the case that they are parallel:



Method of Lagrange multipliers: economical interpretation

- ✓ In economics the optimal profit to a player is calculated subject to a constrained space of actions, where a Lagrange multiplier *is the change in the optimal value of the objective function (profit) due to the relaxation of a given constraint*

$$\nabla f(x,y) = \lambda \cdot \nabla g(x,y)$$

⇩

$$\frac{\partial L(x,y)}{\partial g(x,y)} = \lambda$$

in such a context λ is the marginal cost of the constraint, and is referred as the shadow price

Karush–Kuhn–Tucker conditions

- ✓ The Karush–Kuhn–Tucker (KKT) conditions *are first order necessary conditions for a solution in nonlinear programming to be optimal*, provided that some regularity conditions are satisfied.

- ✓ Allowing inequality constraints, the KKT approach applied to nonlinear programming generalizes the method of Lagrange multipliers, which allows only equality constraints.

Karush–Kuhn–Tucker conditions

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- Let us consider the problem:

$$\min F(x) \tag{1.1}$$

$$s.t. \quad g_i(x) \leq 0 \quad (\lambda_i) \qquad \forall i = 1, \dots n \tag{1.2}$$

$$h_i(x) = 0 \quad (\mu_i) \qquad \forall j = 1, \dots m \tag{1.3}$$

- For this problem, the KKT conditions are:

$$\nabla f(x) + \sum_{i=1}^{n} \lambda_i \nabla \mathrm{g}_i(\mathrm{x}) + \sum_{j=1}^{n} \mu_i \nabla \mathrm{h}_j(\mathrm{x}) = 0 \tag{1.4}$$

$$0 \geq g_i(x) \perp \lambda_i \geq 0 \qquad \forall i = 1, \dots n \tag{1.5}$$

$$0 = h_i(x) \qquad \mu_j \text{ free} \qquad \forall j = 1, \dots m \tag{1.6}$$

The solution stationarity is ensured by the equation (1.4). Equations (1.5) and (1.6) ensure complementarity and feasibility of a solution

Source: [1]

Introduction to MCP formulations

Let us provide the following illustration of such a mathematical structure based on a simple problem faced by a gas producer:

$$\max_{q \geq 0} \Pi = qp(q) - C(q) \tag{1.7}$$

$$s.t.\, q \leq Q \tag{1.8}$$

where:

q - gas sales
p(q) - affine inverse demand function
C(q) – production cost function

Introduction to MCP formulations

The KKT conditions for this problem are:

$$0 \leq q \perp p + (\frac{\partial p}{\partial q} q) - C'(q) + \lambda \leq 0 \qquad (1.9)$$

$$0 \leq \lambda \perp (q - Q) \leq 0 \qquad (1.10)$$

Equation (1.9) is a short way to express the following complementarity problem:

$$0 \leq q$$

$$p - C'(q) + \lambda \leq 0$$

$$q(p - C'(q) + \lambda) = 0$$

where symbol ⊥ states orthogonality

Introduction to MCP formulations

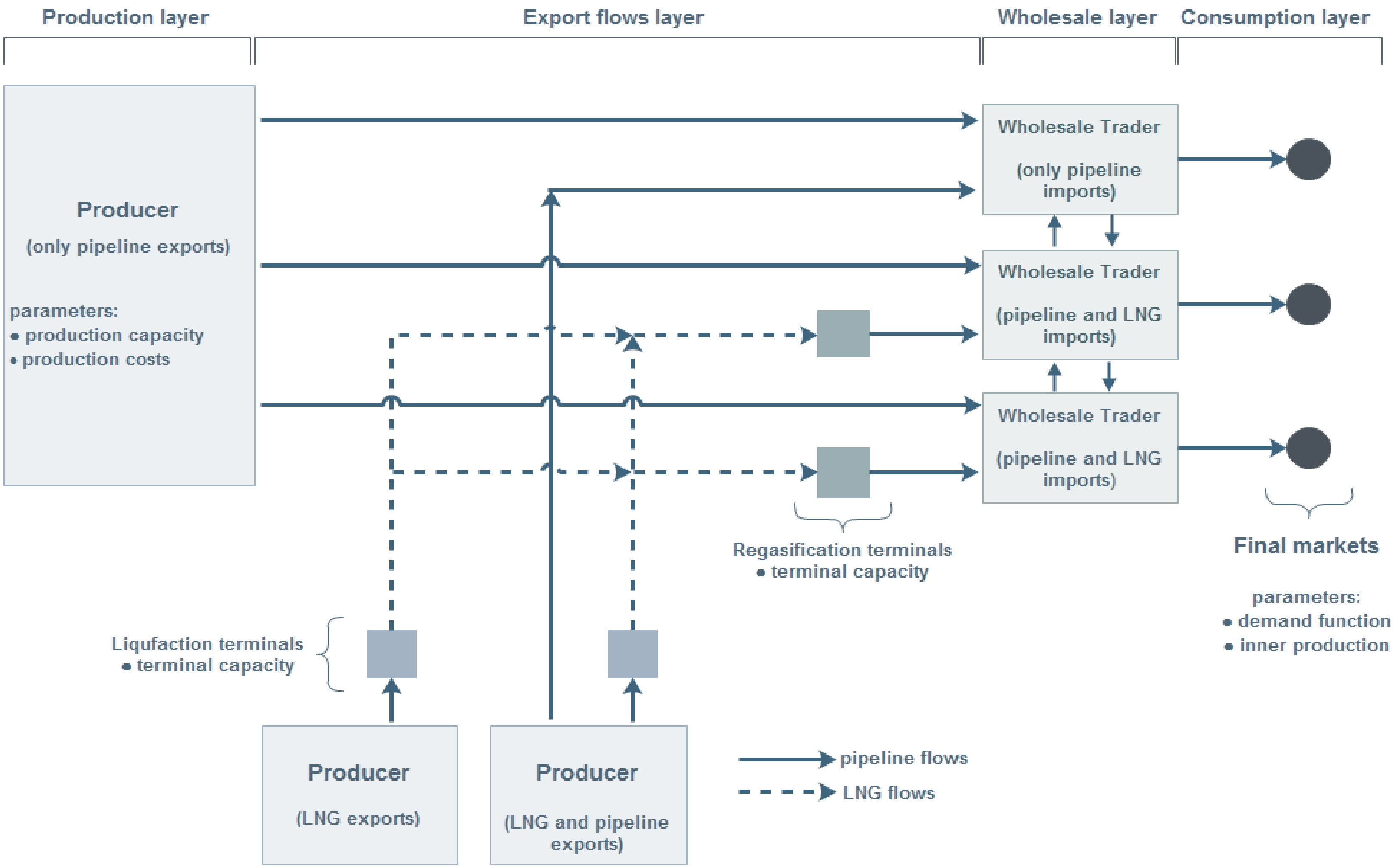
- ✓ The complementarity model of a market is done by combining the KKTs of all market players with market clearing conditions.
- ✓ Numerical problems in MCP format can be efficiently solved with PATH solver by using the GAMS software.
- ✓ The General Algebraic Modelling System (GAMS) is a modeling system used for mathematical programming and optimization. GAMS is designed to model complex and large-scale problems, such as: LP, NLP, MIP, MINLP, etc.

Contents:

Network representation

- ✓ Market participants as producers, wholesale traders, final consumers, LNG terminals are represented in the model as nodes (*N*)
- ✓ There is a list of activities possible to happen in certain node accordingly to its geographical location: *production, export, import, consumption*
- ✓ All nodes in the model are interconnected through arcs. Data for the existing European gas infrastructure was taken from ENTSOG. Arcs have exogenously assigned capacity $cap_{n,m}^{pipe}$ in bcm/a
- ✓ Pipeline interconnections are modelled only by one-directional arcs, although transmission pipelines theoretically could be bidirectional. Gas flows which have to be feasible in two directions are achieved via two one-directional arcs
- ✓ The model neglects gas friction and pressure drops in the network

Structure



Production layer
Export flows layer
Wholesale layer
Consumption layer
Producer
(only pipeline exports)
parameters:
• production capacity
• production costs
Wholesale Trader
(only pipeline imports)
Wholesale Trader
(pipeline and LNG imports)
Wholesale Trader
(pipeline and LNG imports)
Regasification terminals
• terminal capacity
Final markets
parameters:
• demand function
• inner production
Liqufaction terminals
• terminal capacity
Producer
(LNG exports)
Producer
(LNG and pipeline exports)
pipeline flows
LNG flows

Set of assumptions

- ✓ No certain gas flow destination, which gives each consumer the possibility to choose own supplier independently
- ✓ Gas producers have full information about the demand in each node and adjust their production amount optimizing their profits
- ✓ The model operates with an assumption of one wholesale trader located in each country and the absence of vertical integration between companies on subsequent layers
- ✓ The model is based on a static modelling approach. Thus, we exogenously assign investments in natural gas infrastructure (such as new pipeline or LNG capacity which enters the model in expected year of completion)

Contents:

Mathematical description: Producer

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- ✓ Producer's objective is to maximize its profit ($\prod_n^{prod}$) by deciding the quantity of gas to be produced (pr)
- ✓ Its profit results from selling the gas produced at the price ($PtoE_{p,n}$) minus his production costs ($PrC_{p,n}$) which is a linear function from the quantity of gas produced

$$\max_{pr_{p,n}} \Pi_n^{prod} = \underbrace{(pr_{p,n} \cdot PtoE_{p,n})}_{\text{profit}} - \underbrace{PrC_{p,n}(pr_{p,n}),}_{\text{production costs}} \qquad \forall p, n \tag{1}$$

where (1) represents the producer's profit function

Mathematical description: Producer

Each producer operates under the production capacity constraint:

$$cap_n^{prod} \geq \sum_p pr_{p,n} \quad (\lambda_n^{prod}), \quad \forall n \tag{1.2}$$

$$pr_{p,n} \geq 0, \quad \forall p,n \tag{1.3}$$

where equation (1.2) is a production constraint and (1.3) ensures that production variable takes only positive values

Mathematical description: Producer

✓ Following the common approach of obtaining a complementarity model, the problem has to be converted into a minimization problem. The signs of the objective function will be inverted and constraints restructured to the following form:

$$\min_{pr_{p,n}} -\Pi_n^{prod} = -pr_{p,n} \cdot PtoE_{p,n} + PrC_{p,n}(pr_{p,n}), \quad \forall p, n \tag{1.4}$$

$$subject\ to$$

$$cap_n^{prod} - \sum_p pr_{p,n} \geq 0 \quad (\lambda_n^{prod}), \quad \forall n \tag{1.5}$$

$$pr_{p,n} \geq 0, \quad \forall p, n \tag{1.6}$$

Mathematical description: Producer

✓ By deriving the first-order conditions (FOCs) we obtain Karush-Kuhn-Tucker conditions for this problem:

$$\forall p,n: \qquad 0 \leq pr_{p,n} \perp \left(-PtoE_{p,n} + \frac{\partial PrC_{p,n}(pr_{p,n})}{\partial pr_{p,n}} + \lambda_n^{prod}\right) \geq 0 \qquad (1.6)$$

$$\forall n: \qquad 0 \leq \lambda_n^{prod} \perp \left(cap_n^{prod} - \sum_p pr_{p,n}\right) \geq 0 \qquad (1.7)$$

Linear cost function ensures convexity requirement of KKT conditions to find an optimal solution for the given problem

Mathematical description: Exporter

- ✓ The objective of an exporter is to maximize its profit ($\Pi^{exp}_{n,m}$) from gas sales to the market *m* at the border price (bp_m)
- ✓ Natural gas has also to be transported to a node *m;* ($tfee_{n,m}$) is a transport fee paid by an exporter to use arc(s) between *n* and *m* nodes

$$\max_{\substack{exp_{p,n\to m} \\ expfl_{p,n\to m}}} \quad \Pi^{exp}_{n,m} = \underbrace{(exp_{p,n\to m} \cdot bp_m)}_{\text{gas sales}} - \underbrace{(exp_{p,n\to m} \cdot PtoE_{p,n})}_{\text{purchase expenses}} - \underbrace{\sum_{n\to m \in N} tfee_{n\to m} \cdot expfl_{p,n\to m}}_{\text{transportation expenses}} \qquad (2.1)$$

where equation (2.1) is a represents the profit function of an exporter

Mathematical description: Exporter

Subject to:

$$\left[\sum_{m\neq n} exp_{p,n\rightarrow m} - \sum_{m\neq n} expfl_{p,n\rightarrow m}\right] + \left[\sum_{m\neq n} expfl_{p,m\rightarrow n} - \sum_{m\neq n} exp_{p,m\rightarrow n}\right] = 0\ (\varphi^{exp}_{p,n}), \forall p, n \quad (2.2)$$

$$exp_{p,n\rightarrow m} \geq 0, \quad \forall p, n, m \quad (2.3)$$

$$expfl_{p,n\rightarrow m} \geq 0, \quad \forall p, n, m \quad (2.4)$$

Equation (2.2) is a constraint which ensures flow and activity conservation. Literally it means that trade gas volumes have to be equal to the physical gas flow.

Mathematical description: Exporter

Thus, for an exporter P placed in export node *n,* all sales out of node *n* have to be equal to the physical flows out of this node:

$$\left[\sum_{m\neq n} exp_{p,n\rightarrow m} - \sum_{m\neq n} expfl_{p,n\rightarrow m}\right] = 0, \qquad (\varphi_{p,n}^{exp}) \qquad \forall p,n,m$$

In case we consider *n* node as an import node, all import trades from other nodes to node *n* have to be equal to all physical gas inflows:

$$\left[\sum_{m\neq n} expfl_{p,m\rightarrow n} - \sum_{m\neq n} exp_{p,m\rightarrow n}\right] = 0, \qquad (\varphi_{p,m}^{exp}) \qquad \forall p,n,m$$

Note that each node for some set of arcs can be viewed as import or export node

Mathematical description: Exporter

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✓ Deriving KKTs from the corresponding minimization problem we obtain:

$$\forall p,n,m: \quad 0 \leq exp_{p,n\to m} \perp (-bp_m + PtoE_{p,n} + \varphi_{p,n}^{exp} - \varphi_{p,m}^{exp}) \geq 0 \quad (2.5)$$

$$\forall p,n,m: \quad 0 \leq expfl_{p,m\to n} \perp (tfee_{n\to m} - \varphi_{p,n}^{exp} + \varphi_{p,m}^{exp}) \geq 0 \quad (2.6)$$

$$\forall p,n: \quad \left[\sum_{m\neq n} exp_{p,n\to m} - \sum_{m\neq n} expfl_{p,n\to m}\right] + \left[\sum_{m\neq n} expfl_{p,m\to n} - \sum_{m\neq n} exp_{p,m\to n}\right] = 0 \quad (2.7)$$

Mathematical description: Wholesaler

- ✓ The objective of each trader $(w \in W)$ is to maximize its profit from importing gas from upstream players at (bp_m) and re-selling it to the final market *C*
- ✓ Country of destination can be of the same node where a trader is placed (sales for inner consumption) or a different one (wholesale gas trade).
- ✓ In case of sales to other consumption nodes trader *w* has to pay also transport fee $(tfee_{n,m})$ for using an arc between *n* and *m* nodes.

$$\max_{\substack{whs_{w,n\to m} \\ whfl_{w,n\to m}}} \quad \Pi^{ws}_{n,m} = \underbrace{\left(whs_{w,n\to m} \cdot pFC_n\right)}_{\text{gas sales}} - \underbrace{\left(whs_{w,n\to m} \cdot bp_m\right)}_{\text{purchase expenses}} - \underbrace{\sum_{n\to m \in N} tfee_{n\to m} \cdot whfl_{w,n\to m}}_{\text{transportation expenses}} \tag{3.1}$$

where equation (3.1) is a trader's profit function

Mathematical description: Wholesaler

Subject to:

$$\left[\sum_{m\neq n} whs_{w,n\to m} - \sum_{m\neq n} whfl_{w,n\to m}\right] = 0, \qquad (\varphi^{ws}_{w,n}) \qquad \forall w,n,m \tag{3.2}$$

$$\left[\sum_{m\neq n} whfl_{w,m\to n} - \sum_{m\neq n} whs_{w,m\to n}\right] = 0, \qquad (\varphi^{ws}_{w,m}) \qquad \forall w,n,m \tag{3.3}$$

$$whs_{w,n\to m} \geq 0, \quad \forall w,n,m \tag{3.4}$$

$$whfl_{w,n\to m} \geq 0, \quad \forall w,n,m \tag{3.5}$$

Equations (3.2-3) are constraints which ensure flow and activity conservation.

Mathematical description: Wholesaler

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✓ Deriving KKTs from the corresponding minimization problem we obtain:

$$\forall w,n,m: \quad 0 \leq whs_{w,n\to m} \perp (-pFC_n + bp_m + \varphi_{w,n}^{ws} - \varphi_{w,m}^{ws}) \geq 0 \qquad (3.6)$$

$$\forall w,n,m: \quad 0 \leq whfl_{w,n\to m} \perp (tfee_{n\to m} - \varphi_{w,n}^{ws} + \varphi_{w,m}^{ws}) \geq 0 \qquad (3.7)$$

$$\forall w,n,m: \quad \left[\sum_{m\neq n} whs_{w,n\to m} - \sum_{m\neq n} whfl_{w,n\to m}\right] + \left[\sum_{m\neq n} whfl_{w,m\to n} - \sum_{m\neq n} whs_{w,m\to n}\right] = 0 \qquad (3.8)$$

Mathematical description: TSO

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- ✓ Transmission system operator (TSO) is responsible for allocating network capacity to market players who participate in gas import/export/transit activities
- ✓ TSO uses capacity allocation mechanism which assigns additional network capacity to the player with the highest marginal willingness-to-pay for it, i.e. access to pipeline infrastructure is granted according to those players who value capacity the most

$$\max_{totfl_{n \to m}} \quad \Pi_{n,m}^{TSO} = \underbrace{\left(tfee_{n \to m} \cdot totfl_{n \to m}\right)}_{\text{gas flows allocation}} - \underbrace{TC_{n \to m}\left(totfl_{n \to m}\right)}_{\text{transmission costs}} \qquad \forall n, m \qquad (4.1)$$

It was shown by Cremer et al. (2003) that modelling of profit maximizing competitive TSO gives the same results as social welfare optimization

Mathematical description: TSO

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Subject to:

$$cap_{n\to m}^{pipe} \geq totfl_{n\to m} \quad (\lambda_{n\to m}^{pipe}), \qquad \forall n,m \tag{4.2}$$

$$totfl_{n\to m} \geq 0, \quad \forall n,m \tag{4.3}$$

where *totfl* represents total physical gas flow between two nodes. It equals to a sum of all export and wholesale flows between these nodes:

$$\sum_{m\neq n} expfl_{p,n\to m} + \sum_{m\neq n} whfl_{w,n\to m} = totfl_{n\to m} \quad \left(tfee_{n\to m}\ free\right) \quad \forall n,m \tag{4.4}$$

Where equation (4.1) is the objective function. Constraint (4.2) ensures that the total physical gas flow through the arc $(n-m)$ *will never overcome its capacity.*

Mathematical description: TSO

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- ✓ Transmission costs are linear on the subject of distance over similar terrain [6]. Hence, transmission costs $TC_{n\rightarrow m}(totfl_{n\rightarrow m})$ are assumed to be the product of distances between nodes and average value of LRMC of gas transmission.

- ✓ LRMC includes differentiation on offshore/onshore pipeline and Europe grid/FSU grid. Distances between nodes are assigned exogenously and equal to distances calculated between the centers of the countries relevant to nodes

- ✓ The KKTs from the corresponding minimization problem are:

$$\forall n, m: \quad 0 \leq totfl_{n\rightarrow m} \perp \left(-tfee_{n\rightarrow m} + \frac{\partial TC_{n\rightarrow m}(totfl_{n\rightarrow m})}{\partial totfl_{n\rightarrow m}} + \lambda_{n\rightarrow m}^{pipe}\right) \geq 0 \qquad (4.5)$$

$$\forall w, n, m: \quad 0 \leq \lambda_{n\rightarrow m}^{pipe} \perp \left(cap_{n\rightarrow m}^{pipe} - totfl_{n\rightarrow m}\right) \geq 0 \qquad (4.6)$$

Mathematical description: market clearing

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- ✓ The market clearing equation for the upstream level ensures that the whole quantity of gas produced by producer will be purchased by exporter and sold to the following market level:

$$pr_{p,n} = \sum_{m \neq n} exp_{p,n \to m} \quad (PtoE_{p,n} \;\; free), \quad \forall n, m \qquad (5.1)$$

- ✓ Second clearing condition is satisfied if the entire quantity of gas imported by downstream players equals to the entire gas quantity traded on a wholesale market:

$$\sum_{n} \sum_{p} exp_{p,n \to m} = \sum_{w} \sum_{n} whs_{w,n \to m} \quad (bp_m \;\; free), \quad \forall n, m \qquad (5.2)$$

Mathematical description: market clearing

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- ✓ Equation (4.4) was used to define the total physical gas flow which also serves as a market clearing condition between TSO and exporters/traders:

$$\sum_{m \neq n} expfl_{p,n \to m} + \sum_{m \neq n} whfl_{w,n \to m} = totfl_{n \to m} \quad (tfee_{n \to m} \ free), \quad \forall n, m \qquad (5.3)$$

- ✓ The final market clearing constraint guarantees that the price for final consumers matches the inverse demand function at the equilibrium point:

$$pFC_n - \left(a_n + b_n \cdot \sum_{m \neq n} \sum_{w} whs_{w,m \to n} \right) = 0, \quad \forall n \qquad (5.4)$$

Mathematical description: demand function

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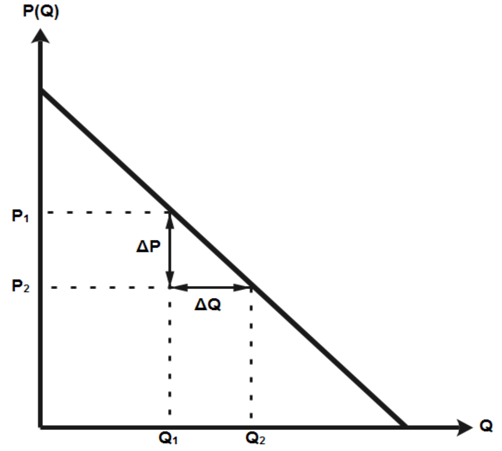
The affine inverse demand function is commonly expressed in the following way:

$$P(Q) = a + b \cdot Q \qquad (6.1)$$

where *P(Q)* represents the price of a good as a function of quantity demanded *(Q).* The constant *b* represents a slope of the function and the constant *a* is an intersection point with the vertical axis.

Inverse demand function is plotted on a coordinate system with the price on the vertical axis and quantity on the horizontal axis:



Mathematical description: demand function

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Estimation of inverse demand function is done around the reference point *(p^{ref}, Q^{ref}):*

$$P^{ref} = a + b \cdot Q^{ref} \tag{6.2}$$

where Q^{ref} is the total consumption in the node *n.* It aggregates consumption quantities of all the final consumers located in that node.

Using definition of the price elasticity of demand (PED), for the demand function the following definitions can be written (here indices are omitted for the sake of simplicity):

$$Q = -\frac{a}{b} + \frac{1}{b} \cdot p; \qquad \varepsilon = -\frac{\partial Q}{\partial p} \cdot \frac{p}{Q} = \frac{1}{b} \cdot \frac{p}{Q};$$

$$b = \frac{p}{Q} \cdot \frac{1}{\varepsilon}; \qquad a = p - b \cdot Q; \tag{6.3}$$

Mathematical description: demand function

Applying results obtained in (6.3) into (1.1) gives the following inverse demand curve:

$$p = P^{ref} - b \cdot Q^{ref} + \frac{P^{ref}}{Q^{ref}} \cdot \frac{1}{\varepsilon} \cdot Q$$

$$p = P^{ref}(1 - \frac{1}{\varepsilon}) + \frac{P^{ref}}{Q^{ref}} \cdot \frac{1}{\varepsilon} \cdot Q$$

$$\Downarrow$$

$$pFC_n - \left(a_n + b_n \cdot \sum_{m \neq n} \sum_{w} whs_{w,m \to n} \right) = 0, \quad \forall n$$

Contents:

Countries included in the model and major open data sources:

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Exporting countries		
Type of connection	pipeline connection	LNG liquefaction terminals
Russia	✔	(✔)
Norway	✔	(✔)
Netherlands	✔	(✔)
Algeria	✔	✔
Libya	✔	(✔)
Eqypt		✔
Nigeria		✔
Qatar		✔
✔	existing connection included in the model	
(✔)	planned/possible connection or terminals not included in the model	

Major sources:

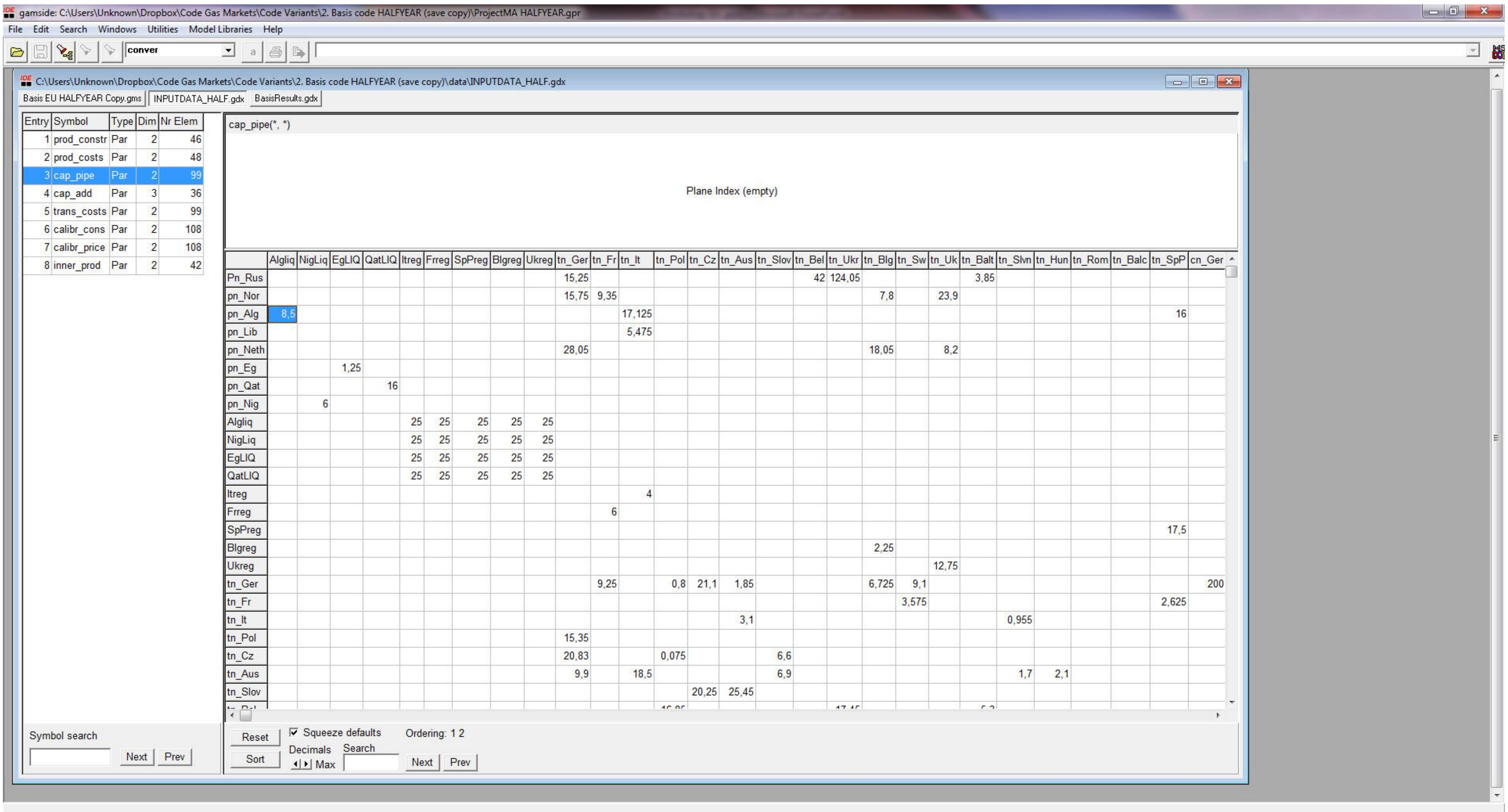
http://www.entsog.eu/

http://www.gie.eu.com/

http://www.naturalgaseurope.com/

Importing countries		
Type of connection	pipeline connection	LNG regasification terminals
Germany	✔	(✔)
France	✔	✔
Italy	✔	✔
Poland	✔	
Czech Rep.	✔	
Austria	✔	
Slovakia	✔	
Belarus	✔	
Ukraine	✔	
Belgium	✔	✔
Switzerland	✔	
UK	✔	✔
Baltic reg.	✔	(✔)
Slovenia	✔	
Hungary	✔	
Romania	✔	
Balkan reg.	✔	(✔)
Spain/Portugal	✔	✔

Data->Math->Modelling->Results

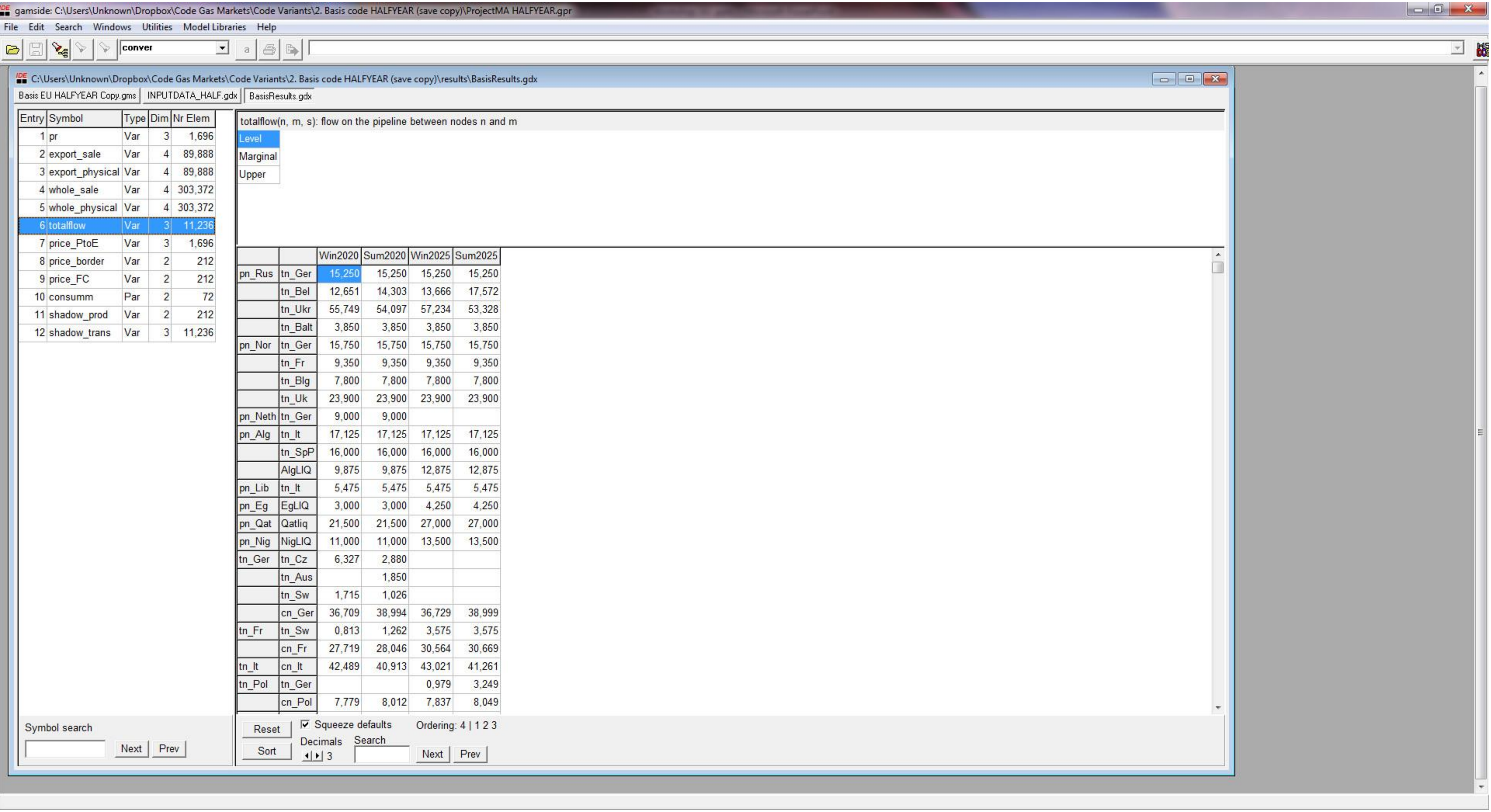


Entry	Symbol	Type	Dim	Nr Elem
1	prod_constr	Par	2	46
2	prod_costs	Par	2	48
3	cap_pipe	Par	2	99
4	cap_add	Par	3	36
5	trans_costs	Par	2	99
6	calibr_cons	Par	2	108
7	calibr_price	Par	2	108
8	inner_prod	Par	2	42

	Algliq	NigLiq	EgLIQ	QatLIQ	Itreg	Frreg	SpPreg	Blgreg	Ukreg	tn_Ger	tn_Fr	tn_It	tn_Pol	tn_Cz
Pn_Rus										15,25				
pn_Nor										15,75	9,35			
pn_Alg	8,5											17,125		
pn_Lib												5,475		
pn_Neth										28,05				
pn_Eg			1,25											
pn_Qat				16										
pn_Nig		6												
Algliq					25	25	25	25	25					
NigLiq					25	25	25	25	25					
EgLIQ					25	25	25	25	25					
QatLIQ					25	25	25	25	25					
Itreg												4		
Frreg											6			
SpPreg														
Blgreg														
Ukreg														
tn_Ger											9,25		0,8	21,1
tn_Fr														
tn_It														
tn_Pol										15,35				
tn_Cz										20,83			0,075	
tn_Aus										9,9		18,5		
tn_Slov														20,25

	tn_Aus	tn_Slov	tn_Bel	tn_Ukr	tn_Blg	tn_Sw	tn_Uk	tn_Balt	tn_Slvn	tn_Hun	tn_Rom	tn_Balc	tn_SpP	cn_Ger
Pn_Rus			42	124,05				3,85						
pn_Nor					7,8		23,9							
pn_Alg													16	
pn_Lib														
pn_Neth					18,05		8,2							
pn_Eg														
pn_Qat														
pn_Nig														
Algliq														
NigLiq														
EgLIQ														
QatLIQ														
Itreg														
Frreg														
SpPreg													17,5	
Blgreg					2,25									
Ukreg							12,75							
tn_Ger	1,85				6,725	9,1								200
tn_Fr						3,575							2,625	
tn_It	3,1								0,955					
tn_Pol														
tn_Cz		6,6												
tn_Aus		6,9							1,7	2,1				
tn_Slov	25,45													

Data->Math->Modelling->Results (GDX)



Entry	Symbol	Type	Dim	Nr Elem
1	pr	Var	3	1,696
2	export_sale	Var	4	89,888
3	export_physical	Var	4	89,888
4	whole_sale	Var	4	303,372
5	whole_physical	Var	4	303,372
6	totalflow	Var	3	11,236
7	price_PtoE	Var	3	1,696
8	price_border	Var	2	212
9	price_FC	Var	2	212
10	consumm	Par	2	72
11	shadow_prod	Var	2	212
12	shadow_trans	Var	3	11,236

		Win2020	Sum2020	Win2025	Sum2025
pn_Rus	tn_Ger	15,250	15,250	15,250	15,250
	tn_Bel	12,651	14,303	13,666	17,572
	tn_Ukr	55,749	54,097	57,234	53,328
	tn_Balt	3,850	3,850	3,850	3,850
pn_Nor	tn_Ger	15,750	15,750	15,750	15,750
	tn_Fr	9,350	9,350	9,350	9,350
	tn_Blg	7,800	7,800	7,800	7,800
	tn_Uk	23,900	23,900	23,900	23,900
pn_Neth	tn_Ger	9,000	9,000		
pn_Alg	tn_It	17,125	17,125	17,125	17,125
	tn_SpP	16,000	16,000	16,000	16,000
	AlgLIQ	9,875	9,875	12,875	12,875
pn_Lib	tn_It	5,475	5,475	5,475	5,475
pn_Eg	EgLIQ	3,000	3,000	4,250	4,250
pn_Qat	Qatliq	21,500	21,500	27,000	27,000
pn_Nig	NigLIQ	11,000	11,000	13,500	13,500
tn_Ger	tn_Cz	6,327	2,880		
	tn_Aus		1,850		
	tn_Sw	1,715	1,026		
	cn_Ger	36,709	38,994	36,729	38,999
tn_Fr	tn_Sw	0,813	1,262	3,575	3,575
	cn_Fr	27,719	28,046	30,564	30,669
tn_It	cn_It	42,489	40,913	43,021	41,261
tn_Pol	tn_Ger			0,979	3,249
	cn_Pol	7,779	8,012	7,837	8,049

Data->Math->Modelling->Results (Example)

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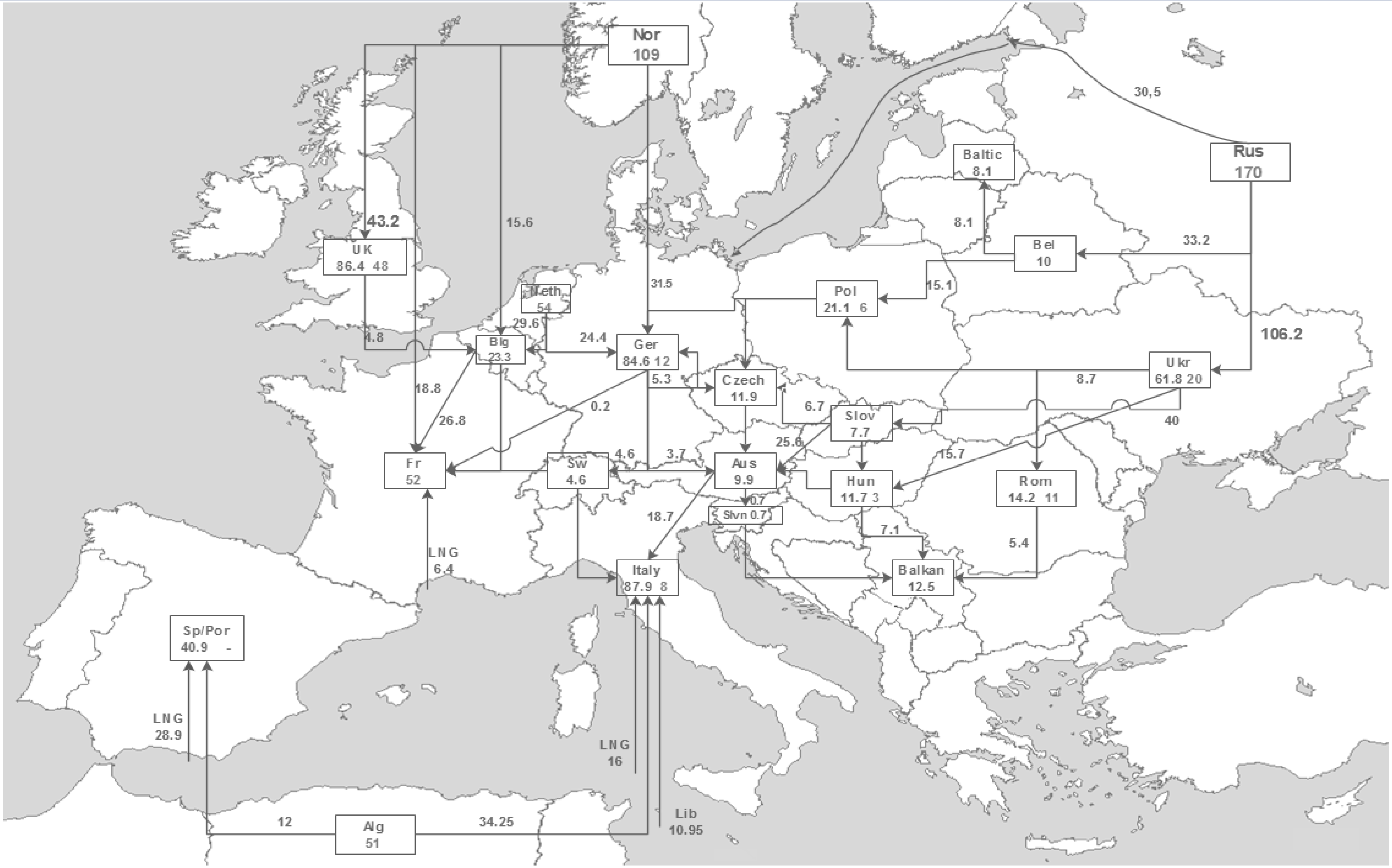

Data->Math->Modelling->Results (Example)

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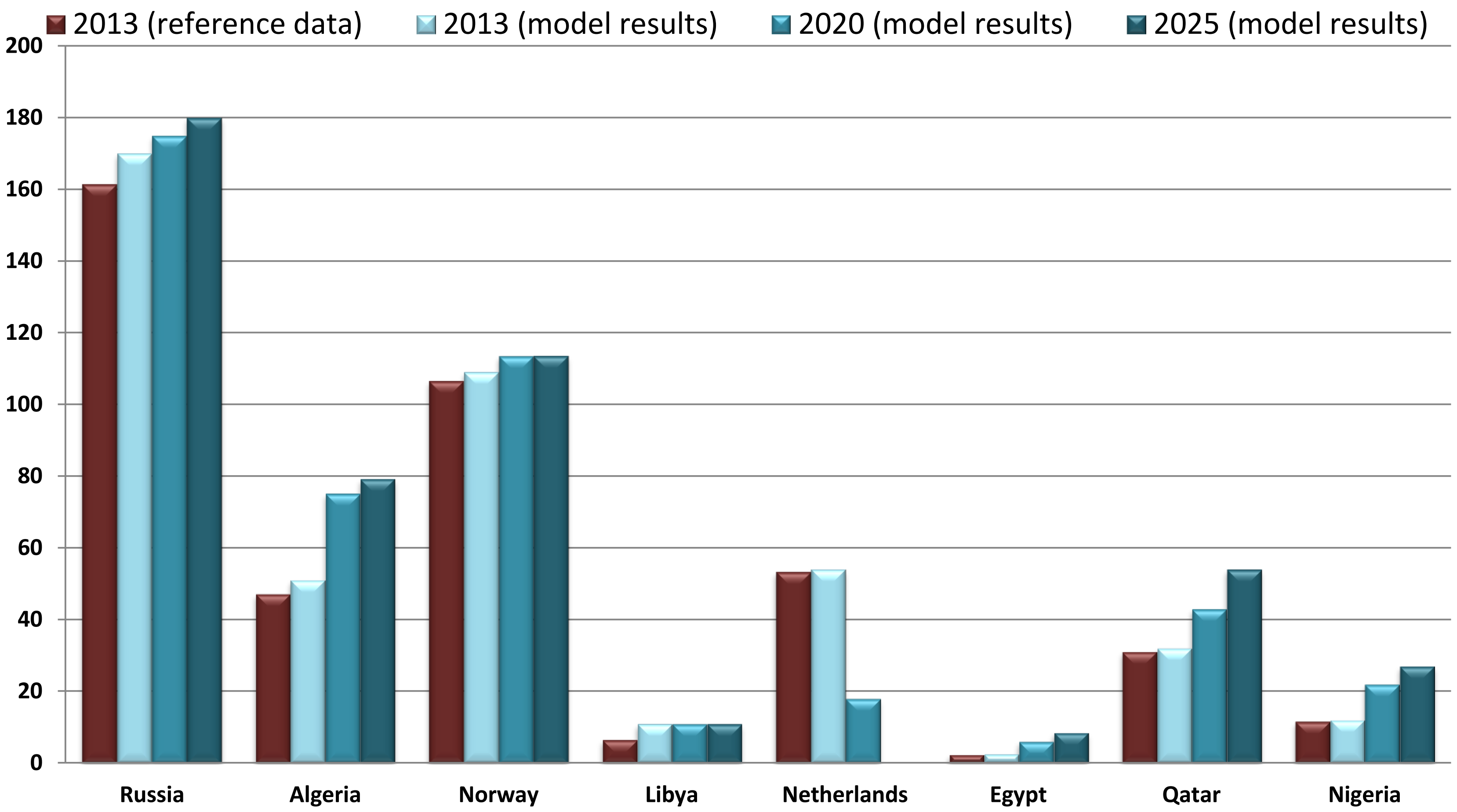

Areas for the future work: middle- and long-term

- **Further work**
  - **Extensive**
    - World Gas Model
    - Gas Storages
  - **Intensive**
    - Non-linear production cost functions
    - Demand sectors
    - Market power
    - Dynamics (shale gas)

THANK YOU!

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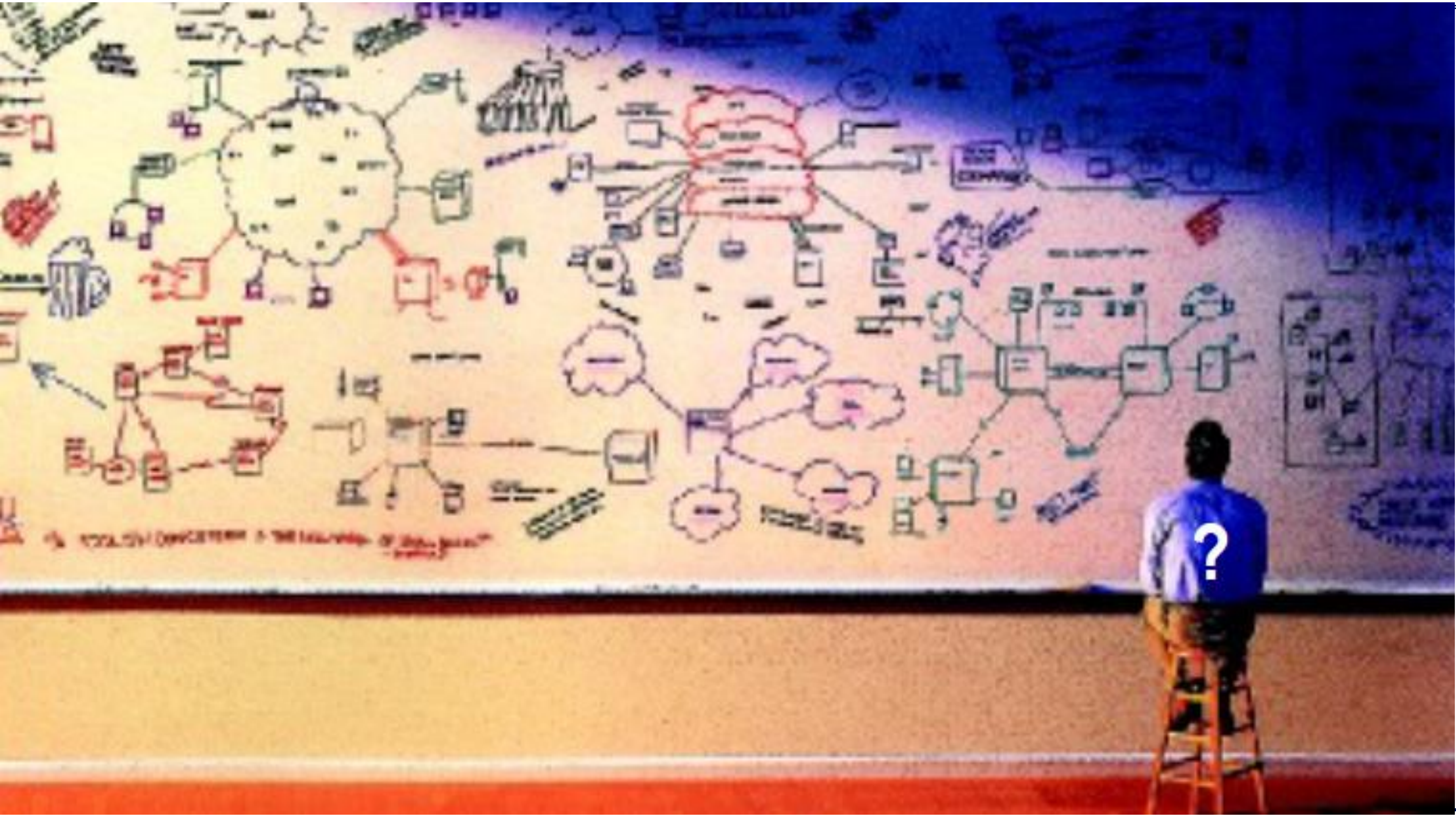

Appendix A: Duality concept

Some (classes of) optimization problems have a "twin" problem,

⇒ This is called the "dual problem"

Illustration using a simple linear program:

Mathematical formulation	Example of interpretation
$\min_{x \in \mathbb{R}} \; a^T x$ $\text{s.t.} \; Ax \leq b \quad (y)$	Minimize cost of supplying electricity subject to engineering and power flow constraints
$\max_{y \in \mathbb{R}_+} \; b^T y$ $\text{s.t.} \; A^T y = a \quad (x)$	Maximize pay-off such that the dual constraints are satisfied

⇒ If the optimal objective values are identical, we call it **strong duality**

Source: [4]

Appendix A: KKT formulations

- The "classical" formulation of Karush-Kuhn-Tucker conditions:

$$0 = \nabla_u f(x^*) + \sum_i \lambda_i^* \nabla_u g_i(x^*) + \sum_j \mu_j^* \nabla_u h_j(x^*) \quad , \quad x_u^* \text{ (free)}$$

$$0 \geq g_i(x^*) \perp \lambda_i^* \geq 0$$

$$0 = h_j(x^*) \quad , \quad \mu_j^* \text{ (free)}$$

- One alternative formulation (of many) :

$$0 \leq \nabla_u f(x^*) + \sum_i \lambda_i^* \nabla_u g_i(x^*) + \sum_j \mu_j^* \nabla_u h_j(x^*) \perp x_u^* \geq 0$$

$$0 \leq -g_i(x^*) \perp \lambda_i^* \geq 0$$

$$0 = h_j(x^*) \quad , \quad \mu_j^* \text{ (free)}$$

Source: [4]

Appendix B: model comparisons

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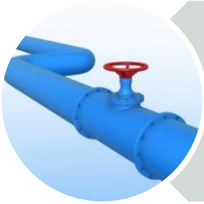
Name	Author	Type	Region	MP	Nodes	Time Scale	Time resolution	Seasons	Dynamics
NEMS	EIA U.S.	LP	North America	no	15	2030	1 year	2	yes
ICF GMM	ICF Int.	NLP	US	no	114	several years	monthly	12	no
WGM	Egging	MCP	World	yes	41	2030	5 years	2	yes
FRISBEE	Statistics Norway	PE	World	no	13	2030	1 year	1	yes
COLUMBUS	Hecking and Panke, EWI	MCP	World	yes	-	2050	monthly	12	yes
GASMOD	Holz, DIW Berlin	MCP	Europe	yes	6	2025	10 years	1	yes
GASTALE	Lise and Hobbs	MCP	Europe	yes	19	2030	5 years	3	yes
TIGER	Lochner et al., EWI	LP	Europe	no	-	2020	monthly	12	yes
NATGAS	Zwart and Mulder	MCP	Europe	yes	-	2035	5 years	2	yes
Current	**BTU LE**	**MCP**	**Europe**	no	**53**	**2025**	**5 years**	**1**	**no**

Appendix C: assumptions for a base case scenario



All gas infrastructure projects (pipeline or LNG capacity extensions) which are at a completion stage will be on operation within the planned time;

Gas production/field depletion of the main gas fields for each producer will follow an expected pattern;

Growth rate of compound annual demand for natural gas (CAGR) will follow projections of IEA (2012) and is assumed to be +0,7% for all countries in the model;

The current conflict in east Ukraine will not have a direct impact on transit politics, i.e. Ukraine will continue to exploit gas infrastructure for transit services; no emergencies happen.

Appendix D: literature used

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1. Bertsekas, D.P., 1999. Nonlinear programming.

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