Mathematical modelling of natural gas market

BTU Cottbus-Senftenberg
Chair of Energy Economics
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Content:

Sets

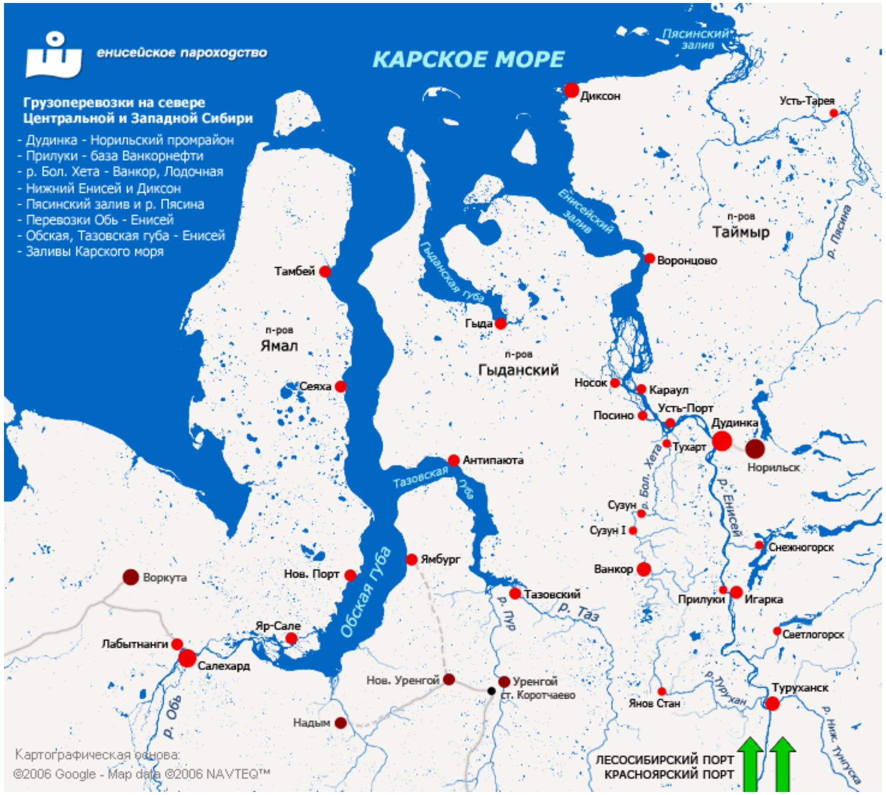
N, M	set of all nodes in the model. *N* includes: *production nodes, wholesale traders, LNG liquefaction and regasification terminals, storage nodes, investor nodes, consumption nodes*
P	set of all gas producers (upstream players) in the model
W	set of all gas traders (downstream players), that imports gas from producers and deliver it to final markets
T, S	set of all time periods

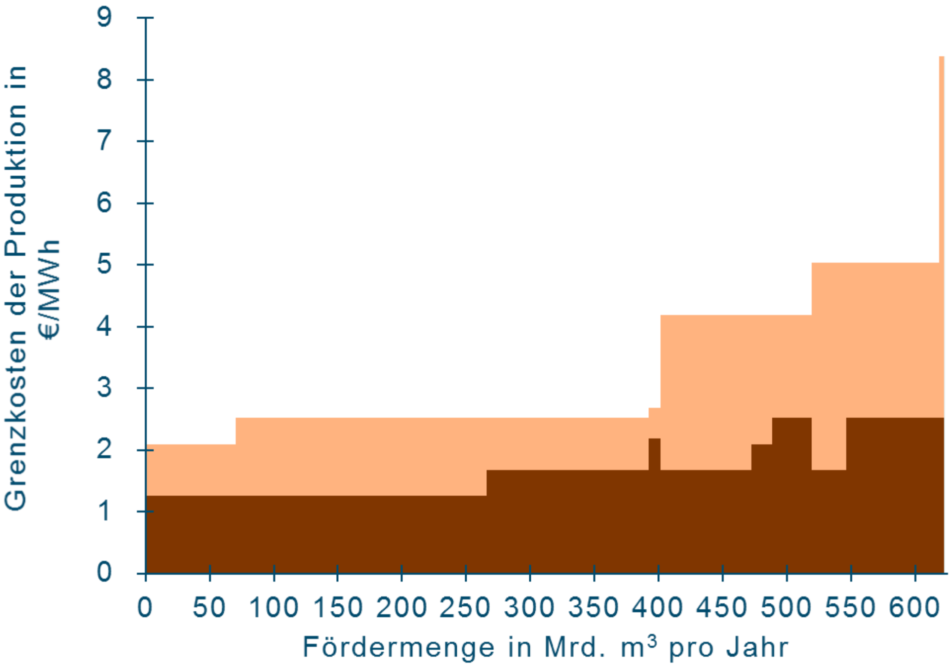
Parameters : production

$prod\_costs_{n,s}^{prod}$	production costs of producer located in a node *n*, $/tcm
$prod\_constr_{p,n,s}^{prod}$	total available production capacity of producer located in a node *n,* bcm

⬛	*Core model*
🟪	*Investor activity*
🟦	*LTC obligation*
🟧	*Storage activity*

Parameters: production





source: IBRD - "The future of NGM in EU"

Parameters: transportation (pipeline & LNG)

$prod\_costs_{n,s}^{prod}$	production costs of producer located in a node *n*, $/tcm
$prod\_constr_{p,n,s}^{prod}$	total available production capacity of producer located in a node *n,* bcm

$cap\_pipe_{n,m}^{pipe}$	initial capacity of connection between *n* and *m* nodes, bcm
$cap\_add_{n,m,s}^{pipe}$	exogenous infrastructure capacity expansion, bcm
$pipe\_constr_{n,m,s}^{pipe}$	sum of *cap_pipe* and *cap_add* parameters, bcm
$trans\_costs_{n,m,s}^{pipe}$	gas transmission cost between *n* and *m* nodes, $/tcm

⬛	*Core model*
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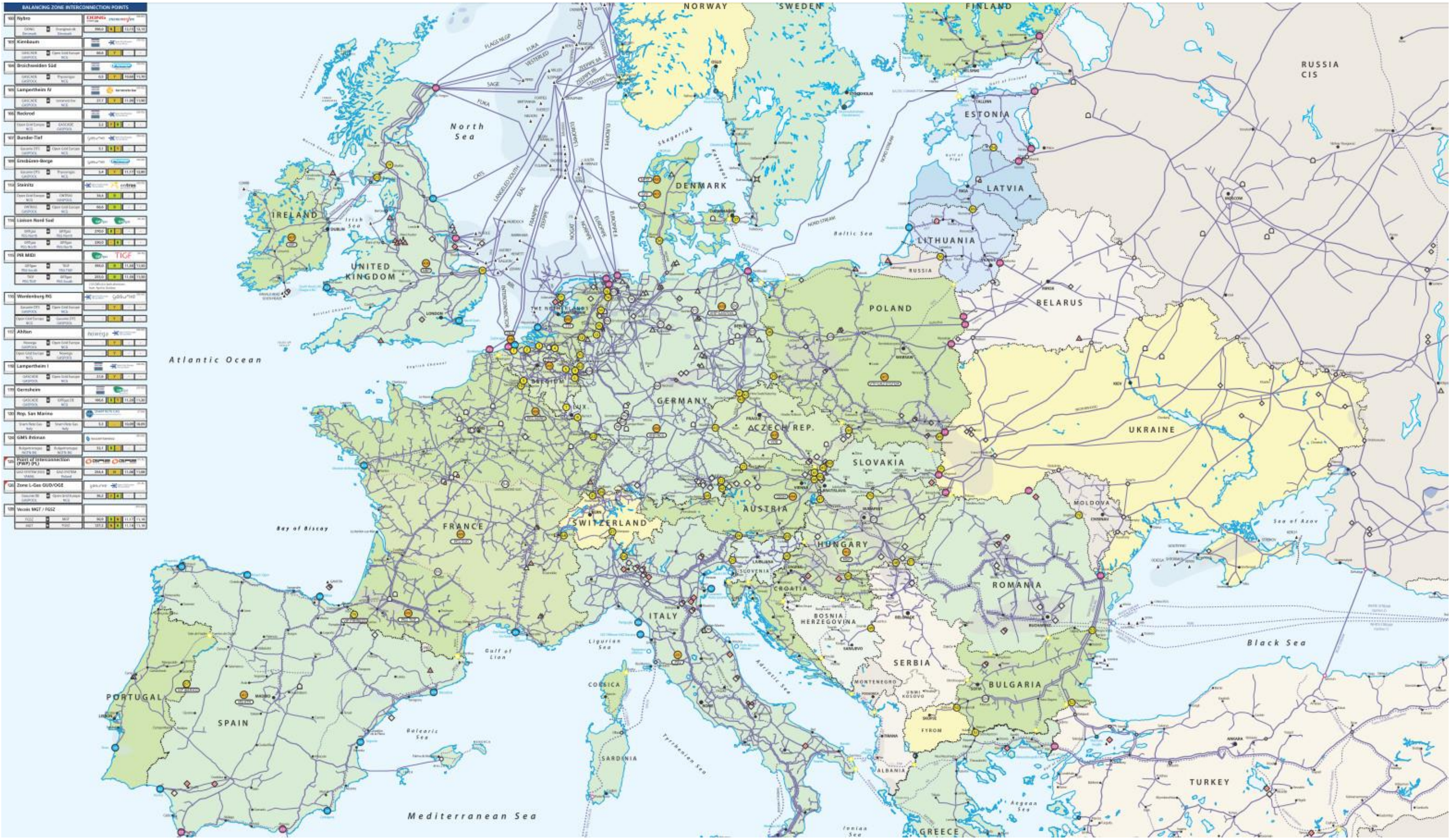
Parameters: transportation (pipeline & LNG)



source: ENTSOG_CAP_MAY2015

Parameters: transportation (pipeline & LNG)

LNG module incorporates:

- ✓ LNG liquefaction and regasification terminals
  - ➢ Installed capacities
  - ➢ Investment plans
- ✓ Geographical location of corresponding harbors and sea distances
- ✓ Shipping cost own estimation based on
  - ➢ Shipping distance
  - ➢ Average speed of tankers
  - ➢ Average LNG carrier size
  - ➢ Fuel consumption of LNG vessels
  - ➢ Average harbor costs, etc.

LNG terminals data:
GIE LNG MAP 2015

Sea distances calculation:
http://www.sea-distances.org

Parameters: transportation (pipeline & LNG)

$prod\_costs_{n,s}^{prod}$	production costs of producer located in a node *n*, $/tcm
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⬛	*Core model*
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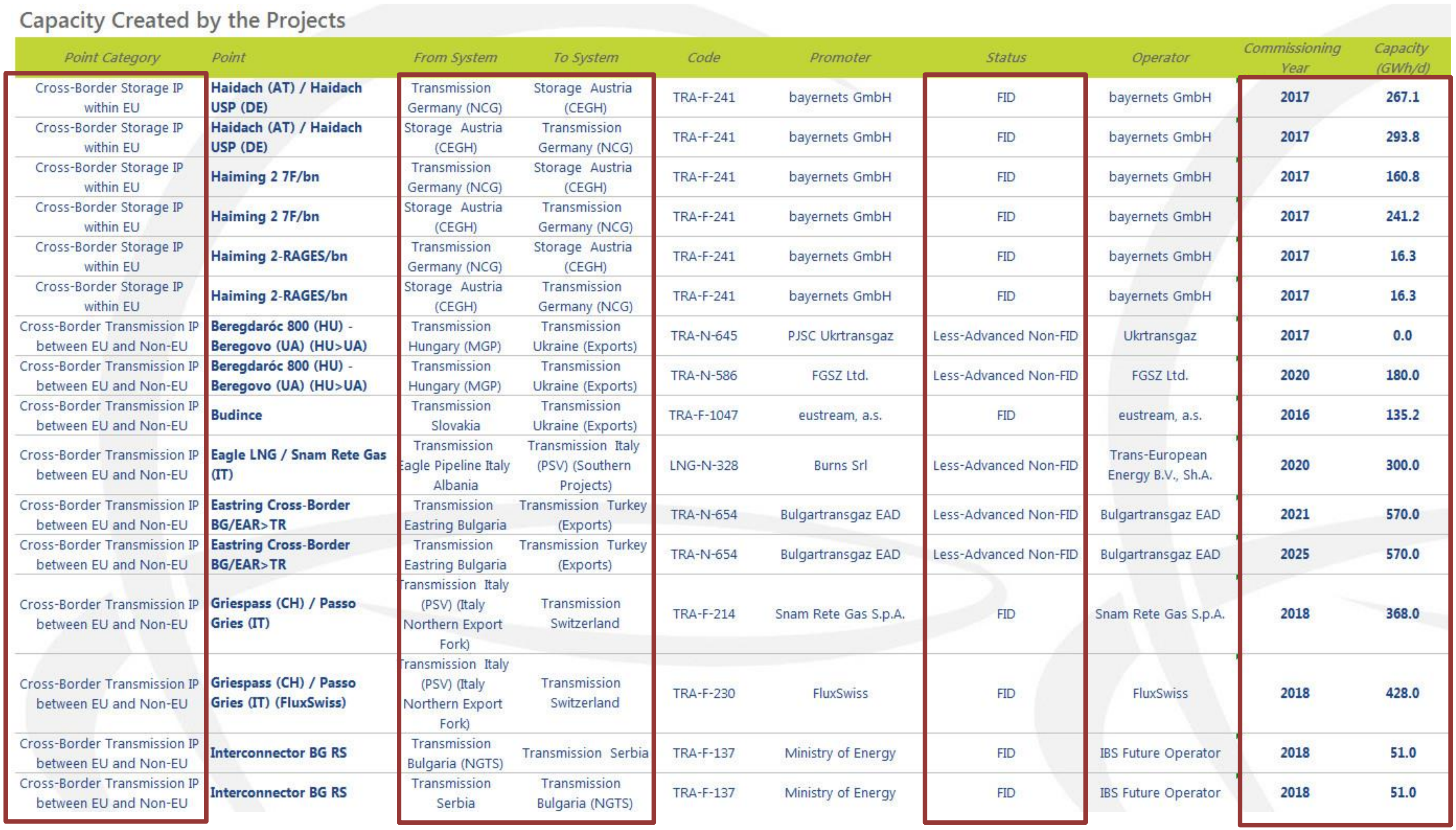
Parameters: semi-automatic exogenous capacity extensions (pipeline & LNG & storage)



Capacity Created by the Projects

Point Category	Point	From System	To System	Code	Promoter	Status	Operator	Commissioning Year	Capacity (GWh/d)
Cross-Border Storage IP within EU	**Haidach (AT) / Haidach USP (DE)**	Transmission Germany (NCG)	Storage Austria (CEGH)	TRA-F-241	bayernets GmbH	FID	bayernets GmbH	**2017**	**267.1**
Cross-Border Storage IP within EU	**Haidach (AT) / Haidach USP (DE)**	Storage Austria (CEGH)	Transmission Germany (NCG)	TRA-F-241	bayernets GmbH	FID	bayernets GmbH	**2017**	**293.8**
Cross-Border Storage IP within EU	**Haiming 2 7F/bn**	Transmission Germany (NCG)	Storage Austria (CEGH)	TRA-F-241	bayernets GmbH	FID	bayernets GmbH	**2017**	**160.8**
Cross-Border Storage IP within EU	**Haiming 2 7F/bn**	Storage Austria (CEGH)	Transmission Germany (NCG)	TRA-F-241	bayernets GmbH	FID	bayernets GmbH	**2017**	**241.2**
Cross-Border Storage IP within EU	**Haiming 2-RAGES/bn**	Transmission Germany (NCG)	Storage Austria (CEGH)	TRA-F-241	bayernets GmbH	FID	bayernets GmbH	**2017**	**16.3**
Cross-Border Storage IP within EU	**Haiming 2-RAGES/bn**	Storage Austria (CEGH)	Transmission Germany (NCG)	TRA-F-241	bayernets GmbH	FID	bayernets GmbH	**2017**	**16.3**
Cross-Border Transmission IP between EU and Non-EU	**Beregdaróc 800 (HU) - Beregovo (UA) (HU>UA)**	Transmission Hungary (MGP)	Transmission Ukraine (Exports)	TRA-N-645	PJSC Ukrtransgaz	Less-Advanced Non-FID	Ukrtransgaz	**2017**	**0.0**
Cross-Border Transmission IP between EU and Non-EU	**Beregdaróc 800 (HU) - Beregovo (UA) (HU>UA)**	Transmission Hungary (MGP)	Transmission Ukraine (Exports)	TRA-N-586	FGSZ Ltd.	Less-Advanced Non-FID	FGSZ Ltd.	**2020**	**180.0**
Cross-Border Transmission IP between EU and Non-EU	**Budince**	Transmission Slovakia	Transmission Ukraine (Exports)	TRA-F-1047	eustream, a.s.	FID	eustream, a.s.	**2016**	**135.2**
Cross-Border Transmission IP between EU and Non-EU	**Eagle LNG / Snam Rete Gas (IT)**	Transmission Eagle Pipeline Italy Albania	Transmission Italy (PSV) (Southern Projects)	LNG-N-328	Burns Srl	Less-Advanced Non-FID	Trans-European Energy B.V., Sh.A.	**2020**	**300.0**
Cross-Border Transmission IP between EU and Non-EU	**Eastring Cross-Border BG/EAR>TR**	Transmission Eastring Bulgaria	Transmission Turkey (Exports)	TRA-N-654	Bulgartransgaz EAD	Less-Advanced Non-FID	Bulgartransgaz EAD	**2021**	**570.0**
Cross-Border Transmission IP between EU and Non-EU	**Eastring Cross-Border BG/EAR>TR**	Transmission Eastring Bulgaria	Transmission Turkey (Exports)	TRA-N-654	Bulgartransgaz EAD	Less-Advanced Non-FID	Bulgartransgaz EAD	**2025**	**570.0**
Cross-Border Transmission IP between EU and Non-EU	**Griespass (CH) / Passo Gries (IT)**	Transmission Italy (PSV) (Italy Northern Export Fork)	Transmission Switzerland	TRA-F-214	Snam Rete Gas S.p.A.	FID	Snam Rete Gas S.p.A.	**2018**	**368.0**
Cross-Border Transmission IP between EU and Non-EU	**Griespass (CH) / Passo Gries (IT) (FluxSwiss)**	Transmission Italy (PSV) (Italy Northern Export Fork)	Transmission Switzerland	TRA-F-230	FluxSwiss	FID	FluxSwiss	**2018**	**428.0**
Cross-Border Transmission IP between EU and Non-EU	**Interconnector BG RS**	Transmission Bulgaria (NGTS)	Transmission Serbia	TRA-F-137	Ministry of Energy	FID	IBS Future Operator	**2018**	**51.0**
Cross-Border Transmission IP between EU and Non-EU	**Interconnector BG RS**	Transmission Serbia	Transmission Bulgaria (NGTS)	TRA-F-137	Ministry of Energy	FID	IBS Future Operator	**2018**	**51.0**

Parameters: demand function

$prod\_costs_{n,s}^{prod}$	production costs of producer located in a node *n*, $/tcm
$prod\_constr_{p,n,s}^{prod}$	total available production capacity of producer located in a node *n,* bcm

$cap\_pipe_{n,m}^{pipe}$	initial capacity of connection between *n* and *m* nodes, bcm
$cap\_add_{n,m,s}^{pipe}$	exogenous infrastructure capacity expansion, bcm
$pipe\_constr_{n,m,s}^{pipe}$	sum of *cap_pipe* and *cap_add* parameters, bcm
$trans\_costs_{n,m,s}^{pipe}$	gas transmission cost between *n* and *m* nodes, $/tcm

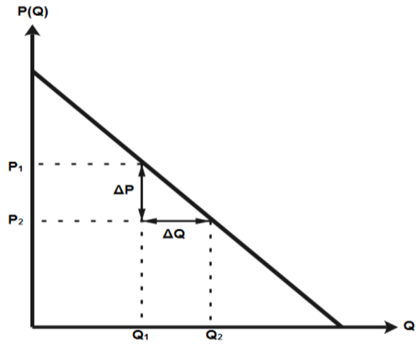
$cons_{n,s}^{ref}$	reference consumption in node *N, bcm*
$price_{n,s}^{ref}$	reference price in node *N,* $/tcm
σ_{n}^{dem}	PED (price elasticity of demand) in market *N*

⬛	*Core model*
🟪	*Investor activity*
🟦	*LTC obligation*
🟧	*Storage activity*

Parameters: demand function

Estimation of inverse demand function is done around the reference point *(P^{ref}, Q^{ref}):*

$$P^{ref} = a + b \cdot Q^{ref}$$



where Q^{ref} is the total consumption in the node *n.* It aggregates consumption quantities of all the final consumers located in that node.

Using definition of the price elasticity of demand (PED), for the demand function the following definitions can be written (here indices are omitted for the sake of simplicity):

$$Q = -\frac{a}{b} + \frac{1}{b} \cdot p; \qquad \varepsilon = -\frac{\partial Q}{\partial p} \cdot \frac{p}{Q} = \frac{1}{b} \cdot \frac{p}{Q};$$

$$b = \frac{p}{Q} \cdot \frac{1}{\varepsilon}; \qquad a = p - b \cdot Q;$$

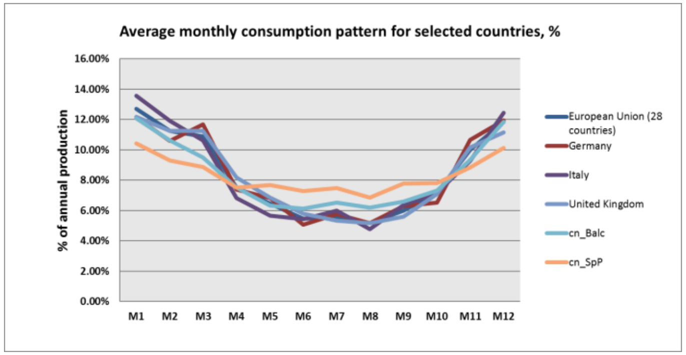
Parameters: demand function

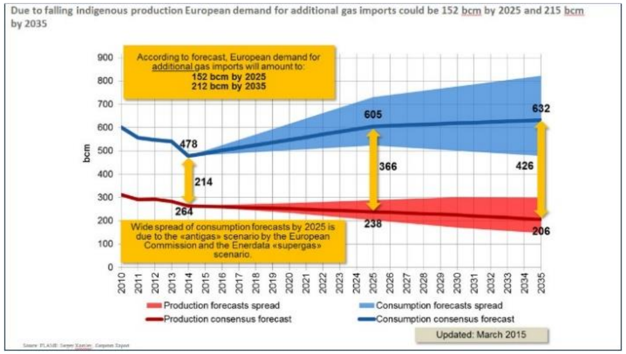
It gives the following inverse demand curve:

$$p = P^{ref} - b \cdot Q^{ref} + \frac{P^{ref}}{Q^{ref}} \cdot \frac{1}{\varepsilon} \cdot Q$$

$$p = P^{ref}(1 - \frac{1}{\varepsilon}) + \frac{P^{ref}}{Q^{ref}} \cdot \frac{1}{\varepsilon} \cdot Q$$

$$p\_FC_{n,s} - \left(\underbrace{a_{n,s} + b_{n,s}}_{\text{Calibration}} \cdot \sum_{w} \sum_{m \neq n} \underbrace{wh\_sale_{w,m,n,s}}_{\text{Consumption (endogenous variable)}} \right) = 0, \quad \forall n, s$$


Average monthly consumption pattern for selected countries, %


Due to falling indigenous production European demand for additional gas imports could be 152 bcm by 2025 and 215 bcm by 2035

According to forecast, European demand for additional gas imports will amount to:
152 bcm by 2025
212 bcm by 2035

Wide spread of consumption forecasts by 2025 is due to the «antigas» scenario by the European Commission and the Enerdata «supergas» scenario.

Updated: March 2015

Parameters

$invLNG\_cap_{n,m,s}^{inv}$	constraint for endogenous investments into gas transport (LNG) infrastructure, bcm
$invLNG\_fixc_{n,m,s}^{inv}$	investor's (LNG) fixed cost, $/tcm

⬛ *Core model*

🟪 *Investor activity*

🟦 *LTC obligation*

🟧 *Storage activity*

Parameters

$invLNG\_cap^{inv}_{n,m,s}$	constraint for endogenous investments into gas transport (LNG) infrastructure, bcm
$invLNG\_fixc^{inv}_{n,m,s}$	investor’s (LNG) fixed cost, $/tcm
$LTC_{p,n,m,s}$	long-term contract obligation closures, bcm

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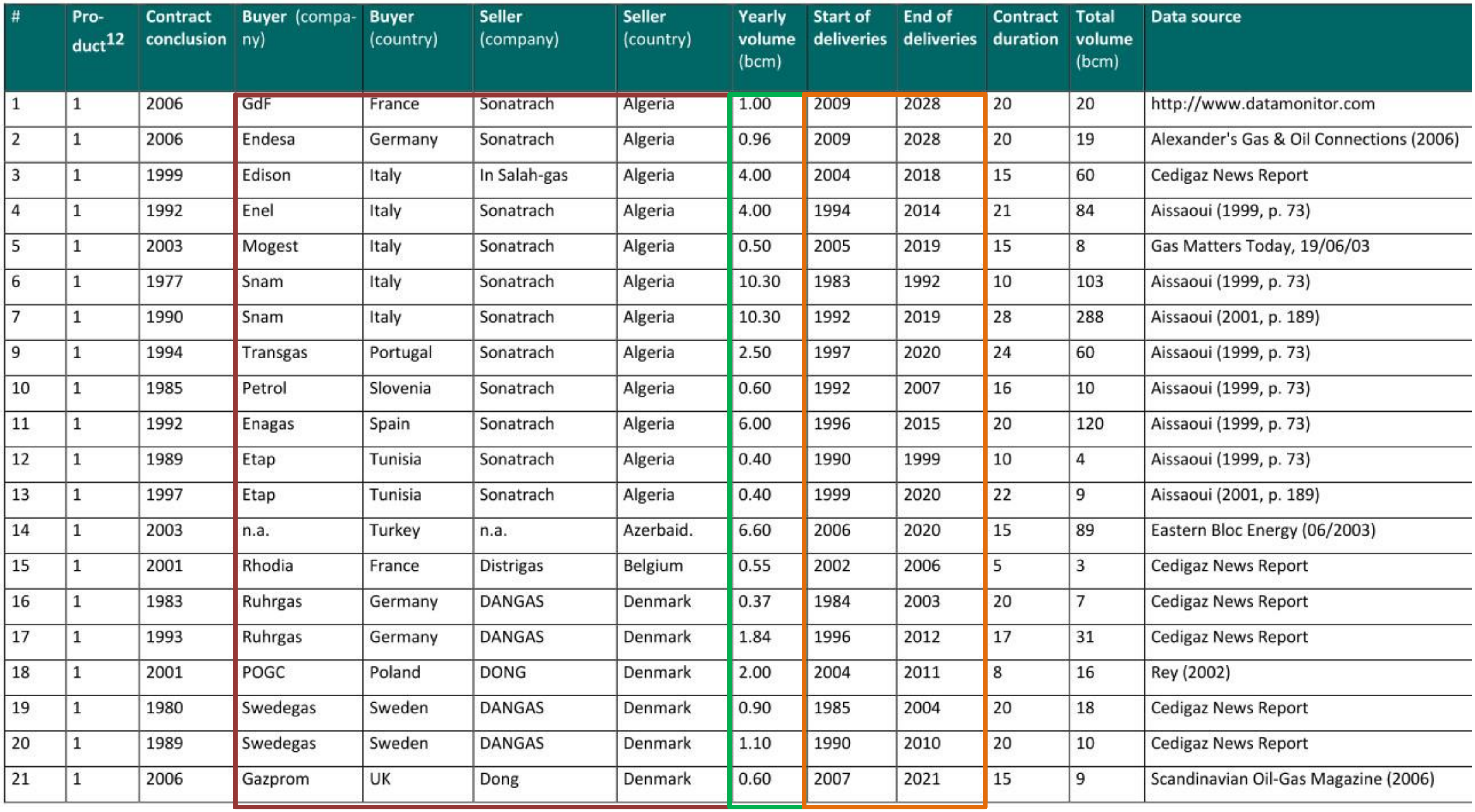
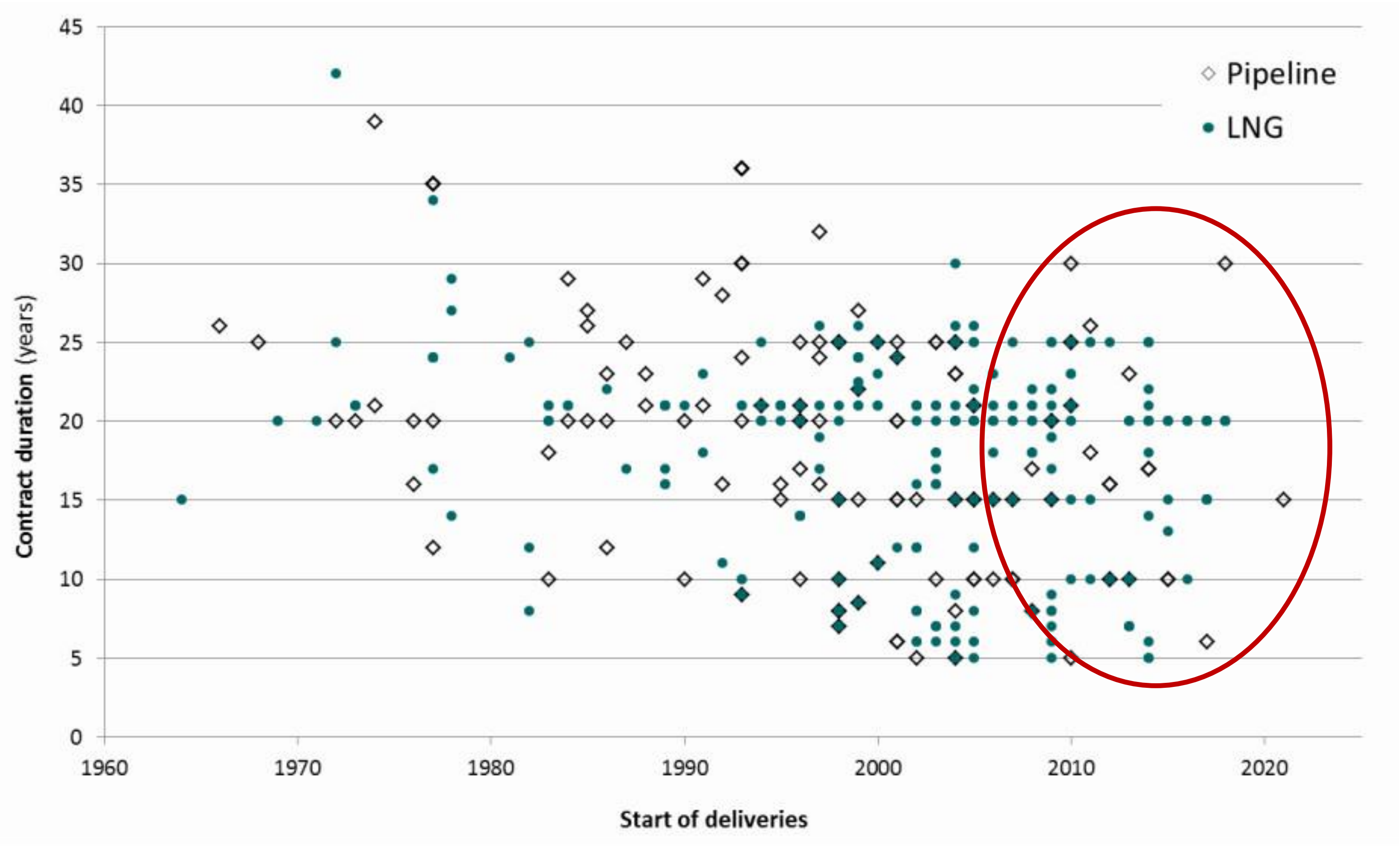
Parameters: long-term contract obligation closures database



#	Product[12]	Contract conclusion	Buyer (company)	Buyer (country)	Seller (company)	Seller (country)	Yearly volume (bcm)	Start of deliveries	End of deliveries	Contract duration	Total volume (bcm)	Data source
1	1	2006	GdF	France	Sonatrach	Algeria	1.00	2009	2028	20	20	http://www.datamonitor.com
2	1	2006	Endesa	Germany	Sonatrach	Algeria	0.96	2009	2028	20	19	Alexander's Gas & Oil Connections (2006)
3	1	1999	Edison	Italy	In Salah-gas	Algeria	4.00	2004	2018	15	60	Cedigaz News Report
4	1	1992	Enel	Italy	Sonatrach	Algeria	4.00	1994	2014	21	84	Aissaoui (1999, p. 73)
5	1	2003	Mogest	Italy	Sonatrach	Algeria	0.50	2005	2019	15	8	Gas Matters Today, 19/06/03
6	1	1977	Snam	Italy	Sonatrach	Algeria	10.30	1983	1992	10	103	Aissaoui (1999, p. 73)
7	1	1990	Snam	Italy	Sonatrach	Algeria	10.30	1992	2019	28	288	Aissaoui (2001, p. 189)
9	1	1994	Transgas	Portugal	Sonatrach	Algeria	2.50	1997	2020	24	60	Aissaoui (1999, p. 73)
10	1	1985	Petrol	Slovenia	Sonatrach	Algeria	0.60	1992	2007	16	10	Aissaoui (1999, p. 73)
11	1	1992	Enagas	Spain	Sonatrach	Algeria	6.00	1996	2015	20	120	Aissaoui (1999, p. 73)
12	1	1989	Etap	Tunisia	Sonatrach	Algeria	0.40	1990	1999	10	4	Aissaoui (1999, p. 73)
13	1	1997	Etap	Tunisia	Sonatrach	Algeria	0.40	1999	2020	22	9	Aissaoui (2001, p. 189)
14	1	2003	n.a.	Turkey	n.a.	Azerbaid.	6.60	2006	2020	15	89	Eastern Bloc Energy (06/2003)
15	1	2001	Rhodia	France	Distrigas	Belgium	0.55	2002	2006	5	3	Cedigaz News Report
16	1	1983	Ruhrgas	Germany	DANGAS	Denmark	0.37	1984	2003	20	7	Cedigaz News Report
17	1	1993	Ruhrgas	Germany	DANGAS	Denmark	1.84	1996	2012	17	31	Cedigaz News Report
18	1	2001	POGC	Poland	DONG	Denmark	2.00	2004	2011	8	16	Rey (2002)
19	1	1980	Swedegas	Sweden	DANGAS	Denmark	0.90	1985	2004	20	18	Cedigaz News Report
20	1	1989	Swedegas	Sweden	DANGAS	Denmark	1.10	1990	2010	20	10	Cedigaz News Report
21	1	2006	Gazprom	UK	Dong	Denmark	0.60	2007	2021	15	9	Scandinavian Oil-Gas Magazine (2006)

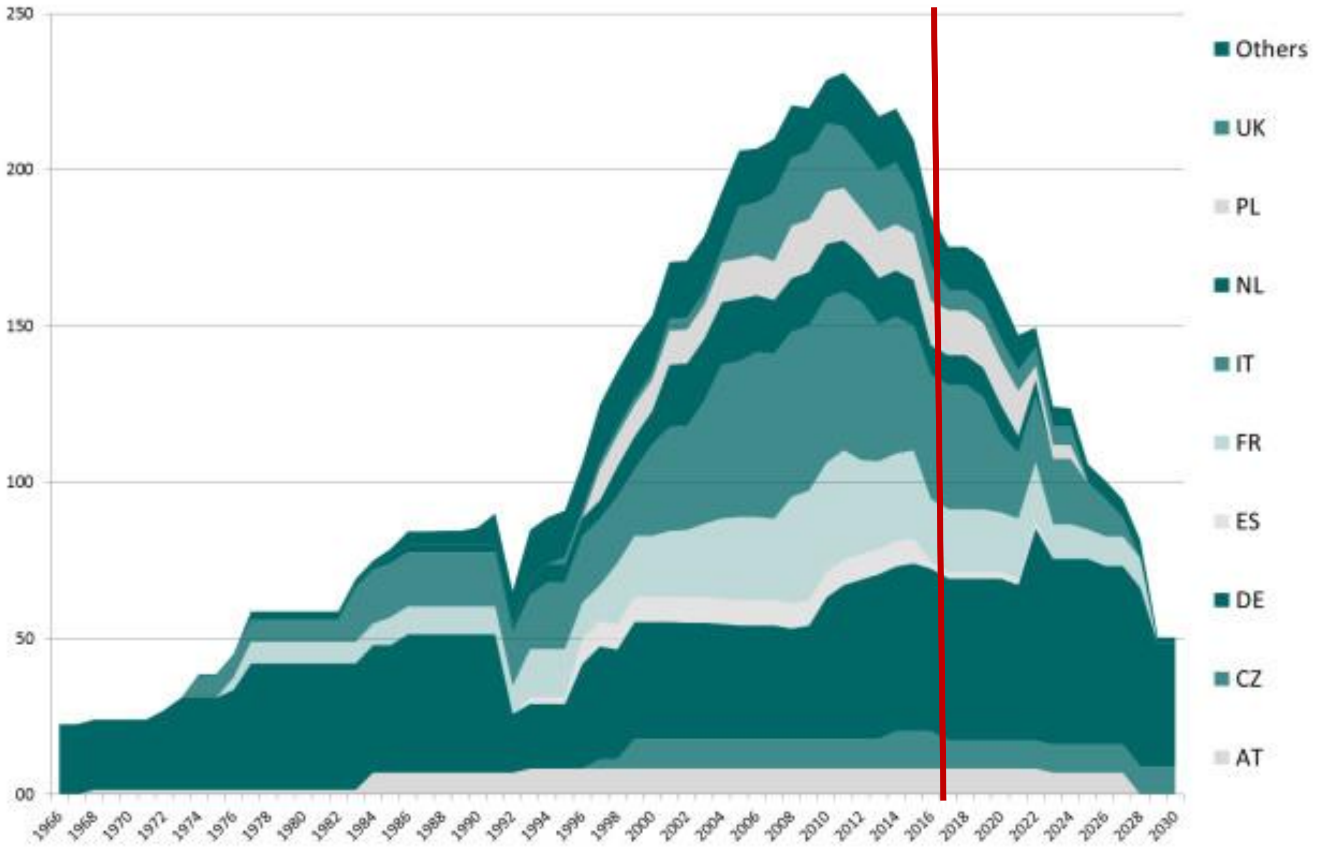
source: DIW “Long-Term Contracts in the Natural Gas Industry – Literature Survey and Data on 426 Contracts (1965-2014)”, 2015

Parameters: long-term contract obligation closures database



source: DIW "Long-Term Contracts in the Natural Gas Industry – Literature Survey and Data on 426 Contracts (1965-2014)", 2015

Parameters: pipeline deliveries covered by the database



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Parameters

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$invLNG\_fixc_{n,m,s}^{inv}$	investor’s (LNG) fixed cost, $/tcm
$LTC_{p,n,m,s}$	long-term contract obligation closures, bcm
$cap\_stor_{n,s}^{st}$	storage capacity located in a node n, bcm
$inj\_vol_{n,s}^{st}$	maximum volume that can be injected into storage facility, bcm per season
$with\_vol_{n,s}^{st}$	maximum volume that can be withdrawed from storage, bcm per season
$loss$	gas losses that occur within one injection/withdrawal cycle, %

⬛	*Core model*
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Variables

$\text{exp}\_sale_{p,n,m,s}$	gas sales of p's exporter from export node *n* to market *m*
$\text{exp}\_ph_{p,n,m,s}$	export physical gas flow from export node *n* to market *m*
$wh\_sale_{w,n,m,s}$	gas sales of w's wholesale trader from market *n* to market *m*
$wh\_ph_{w,n,m,s}$	wholesale gas flow (including transit gas flow) from market to market *m*
$pipe\_fl_{n,m,s}$	total physical gas flow between *n* and *m* nodes
$s\_prod_{p,n,s}^{prod}$	shadow variable of production constraint in production node *p*
$s\_trans_{n,m,s}^{pipe/lng}$	shadow variable of transit constraint for an arc between *n* and *m* nodes
$s\_expbal_{p,n,s}^{exp}$	shadow variable of mass balance constraint for each exporter
$s\_whbal_{w,n,s}^{whs}$	shadow variable of mass balance constraint for each wholesaler
$p\_b_{n,s}$	border price for gas sales in market *n*
$p\_FC_{m,s}$	price of final consumption in consumption node *c(n)*
$tr\_fee_{n,m,s}$	transport fee for using an arc between *n* and *m* nodes

⬛	*Core model*
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Variables

$InvLNG_{n,m,s}$	investments in LNG infrastructure, bcm
$s\_investLNG_{n,m,s}$	shadow variable of LNG investment constraint

$st_{n,s}$	gas stock level at the beginning of a season, bcm
$sv_{n,s}$	injected volume per season, bcm
$sw_{n,s}$	withdrawed volume per season, bcm
$pcw_{n,s}$	shadow variable of storage withdrawal constraint
$pcv_{n,s}$	shadow variable of storage injection constraint
$pcst_{n,s}$	shadow variable of storage capacity constraint
$\alpha_{n,s}$	shadow variable of storage balance equations

⬛	*Core model*
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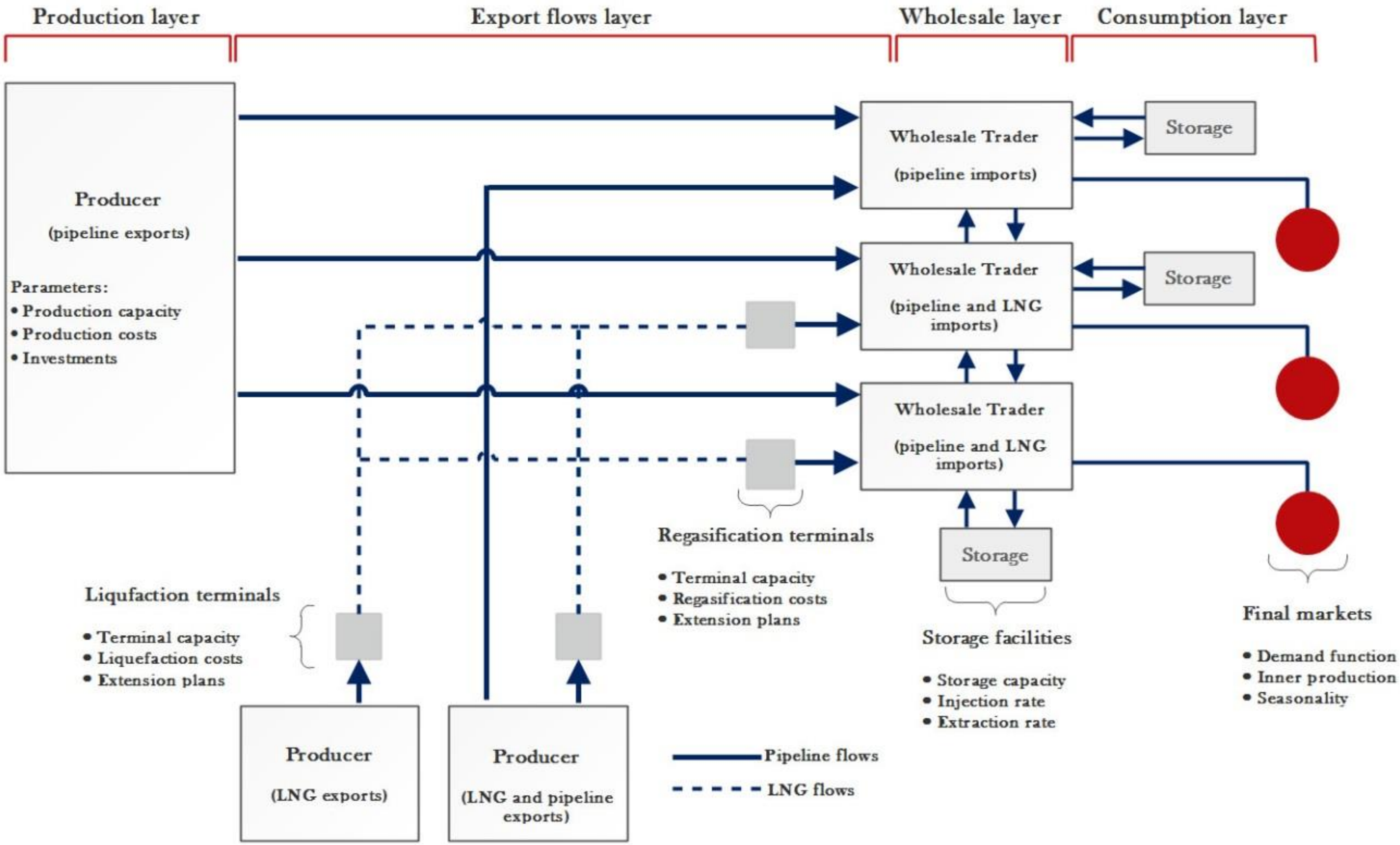
Content:

Brandenburgische
Technische Universität
Cottbus

Model structure: schematic overview



Production layer

Export flows layer

Wholesale layer

Consumption layer

Producer

(pipeline exports)

Parameters:
- Production capacity
- Production costs
- Investments

Wholesale Trader (pipeline imports)

Wholesale Trader (pipeline and LNG imports)

Wholesale Trader (pipeline and LNG imports)

Storage

Storage

Storage

Liqufaction terminals
- Terminal capacity
- Liquefaction costs
- Extension plans

Regasification terminals
- Terminal capacity
- Regasification costs
- Extension plans

Storage facilities
- Storage capacity
- Injection rate
- Extraction rate

Final markets
- Demand function
- Inner production
- Seasonality

Producer (LNG exports)

Producer (LNG and pipeline exports)

Pipeline flows

LNG flows

Supplier problem: objective function

Supplier’s (p∈P) objective is to maximize its profit by deciding the quantity of gas to be produced and exported. Its profit results from selling the gas produced at the border price minus the production and transportation costs. $(tr\_fee_{n,m})$ is a transport fee paid by supplier to use arc(s) between *n* and *m* nodes.

$$\max_{\exp\_sale_{p,n,m,s},\ \exp\_ph_{p,n,m,s}} \Pi_{p,n}^{sup} = \sum_{s}\left\{\sum_{m}\left(\exp\_sale_{p,n,m,s}\cdot p\_b_{m,s} - prod\_costs_{n,s}^{prod}(\exp\_sale_{p,n,m,s})\right) - \sum_{n\to m\in N} tr\_fee_{n,m,s}\cdot \exp\_ph_{p,n,m,s}\right\} \tag{1.1}$$

Supplier problem: constraints

s.t.

$$prod\_constr_{p,n,s}^{prod} - \sum_{m} \exp\_sale_{p,n,m,s} \geq 0, \qquad \forall p,n,s \ \left(s\_prod_{p,n,s}^{prod}\right) \tag{1.2}$$

$$\exp\_sale_{p,n,m,s} - LTC_{p,n,m,s} \geq 0 \qquad \forall p,n,m,s \ \left(s\_ltccap_{p,n,m,s}^{ltc}\right) \tag{x}$$

$$\begin{aligned}&\left[\sum_{m \neq n} \exp\_sale_{p,n,m,s} - \sum_{m \neq n} \exp\_ph_{p,n,m,s}\right] \\ &+ \left[\sum_{m \neq n} \exp\_ph_{p,m,n,s} - \sum_{m \neq n} \exp\_sale_{p,m,n,s}\right] = 0, \qquad \forall p,n,s \ \left(s\_expbal_{p,n,s}^{exp}\right)\end{aligned} \tag{1.3}$$

$$\begin{aligned}&\exp\_sale_{p,n,m,s} \geq 0, \qquad \forall p,n,m,s \\ &\exp\_ph_{p,n,m,s} \geq 0, \qquad \forall p,n,m,s\end{aligned} \tag{1.4}$$

where equation (1.2) is a production constraint, (1.3) ensures flow and trade volume conservation and (1.4) ensures nonnegativity of decision vectors

MCP / Karush–Kuhn–Tucker conditions

The Karush–Kuhn–Tucker (KKT) conditions are first order necessary conditions for a solution in nonlinear programming to be optimal, provided that some regularity conditions are satisfied.

- Let us consider the problem:

$$\min F(x)$$

$$s.t. \quad g_i(x) \leq 0 \quad (\lambda_i) \qquad \forall i = 1, \ldots n \tag{1.5}$$

$$h_i(x) = 0 \quad (\mu_i) \qquad \forall j = 1, \ldots m \tag{1.6}$$

(1.7)

- For this problem, the KKT conditions are:

$$\nabla f(x) + \sum_{i=1}^{n} \lambda_i \nabla g_i(x) + \sum_{j=1}^{m} \mu_i \nabla h_j(x) \geq 0 \perp x \geq 0 \tag{1.8}$$

$$0 \geq g_i(x) \perp \lambda_i \geq 0 \qquad \forall i = 1, \ldots n \tag{1.9}$$

$$0 = h_i(x) \qquad \mu_j \text{ free} \qquad \forall j = 1, \ldots m \tag{1.10}$$

The solution stationarity is ensured by the equation (1.8). Equations (1.9) and (1.10) ensure complementarity and feasibility of a solution

Supplier problem: KKT conditions

The KKTs from the corresponding minimization problem are:

$$0 \leq \exp\_sale_{p,n,m,s}$$

$$\perp -p\_b_{m,s} + prod\_costs_{n,s}^{prod} + s\_prod_{p,n,s}^{prod} + s\_expbal_{p,n,s}^{exp} - s\_expbal_{p,m,s}^{exp} \geq 0 \qquad \forall p, n, m, s$$

(1.11)

$$0 \leq \exp\_ph_{p,n,ms} \perp tr\_fee_{n,m,s} - s\_expbal_{p,n,s}^{exp} + s\_expbal_{p,m,s}^{exp} \geq 0 \qquad \forall p, n, m, s$$

(1.12)

$$0 \leq s\_prod_{p,n,s}^{prod} \perp prod\_constr_{p,n,s}^{prod} - \sum_{m} \exp\_sale_{p,n,m,s} \geq 0 \qquad \forall p, n, s$$

(1.13)

$s\_expbal_{p,n,s}^{exp}$ free,

$$\left[\sum_{m \neq n} \exp\_sale_{p,n,m,s} - \sum_{m \neq n} \exp\_ph_{p,n,m,s}\right] + \left[\sum_{m \neq n} \exp\_ph_{p,m,n,s} - \sum_{m \neq n} \exp\_sale_{p,m,n,s}\right] = 0 \qquad \forall p, n, s$$

(1.14)

Wholesaler problem: objective function

The objective of each trader (w∈W) is to maximize its profit from importing gas from upstream suppliers and satisfying demand of the final market. Wholesale nodes can be also used for the gas transit. No distribution costs (gas allocation inside a single node) is incorporated, i.e. gas is consumed “somewhere” in the node.

$$\max_{wh\_sale_{w,n,m,s}, wh\_ph_{w,n,m,s}} \Pi_{w,n}^{ws} = \sum_{s} \left\{ \sum_{m} \left(wh\_sale_{w,n,m,s} \cdot p\_FC_{m,s} \right) - \sum_{mm} \left(wh\_sale_{w,m,n,s} \cdot p\_b_{n,s} \right) - \sum_{n \to m \in N} tr\_fee_{n,m,s} \cdot wh\_ph_{w,n,m,s} \right\} \tag{2.1}$$

Wholesaler problem: constraints

s.t.

$$\left[\sum_{m\neq n} wh\_sale_{w,n,m,s} - \sum_{m\neq n} wh\_ph_{w,n,m,s}\right] + \left[\sum_{m\neq n} wh\_ph_{w,m,n,s} - \sum_{m\neq n} wh\_sale_{w,m,n,s}\right] = 0\,, \quad \forall w,n,m,s \quad (s\_whbal^{whs}_{w,n,s}) \tag{2.2}$$

$$wh\_sale_{w,n,m,s} \geq 0, \quad \forall w,n,m,s$$

$$wh\_ph_{w,n,m,s} \geq 0, \quad \forall w,n,m,s \tag{2.3}$$

where equation (2.2) ensures flow and trade volume conservation and (2.3) ensures nonnegativity of decision vectors

Wholesaler problem: KKT conditions

The KKTs from the corresponding minimization problem are:

$$0 \leq wh\_sale_{w,n,m,s} \perp -p\_FC_{m,s} + p\_b_{m,s} + s\_whbal^{whs}_{w,n,s} - s\_whbal^{whs}_{w,m,s} \geq 0 \qquad \forall w, n, m, s$$

(2.4)

$$0 \leq wh\_ph_{w,n,m,s} \perp tr\_fee_{n,m,s} - s\_whbal^{whs}_{w,n,s} + s\_whbal^{whs}_{w,m,s} \geq 0 \qquad \forall w, n, m, s$$

(2.5)

$s_whbal^{whs}_{w,n,s}$free,

$$\left[\sum_{m \neq n} wh\_sale_{w,n,m,s} - \sum_{m \neq n} wh\_ph_{w,n,m,s}\right] + \left[\sum_{m \neq n} wh\_ph_{w,m,n,s} - \sum_{m \neq n} wh\_sale_{w,m,n,s}\right] = 0 \quad \forall w, n, s$$

(2.6)

TSO problem: objective function

Transmission system operator (TSO) is responsible for allocating network capacity to market players who participate in gas import/export activities. TSO is assumed to operate the network efficiently. That means that an access to infrastructure is granted according to those players who value capacity the most (capacity allocation mechanism assigns additional network capacity to the player with the highest marginal willingness-to-pay). It was proved by Cremer et al. (2003) that modelling of profit maximizing competitive TSO gives the same results as social welfare optimization in case of convex optimization problem. Its revenues results from congestion charges in case of pipeline saturation and expenses are the actual network operation costs.

$$\max_{pipe\_fl_{n,m,s}} \Pi^{TSO} = \sum_{s} \left\{ \sum_{n \to m} \left(tr\_fee_{n,m,s} \cdot pipe\_fl_{n,m,s} \right) - tr\_costs^{pipe}_{n,m,s}\left(pipe\_fl_{n,m,s}\right) \right\} \tag{3.1}$$

TSO problem: constraints

s.t.

$$pipe\_constr^{pipe}_{n,m,s} + \sum_{s'<s} InvLNG_{n,m,s} \geq pipe\_fl_{n,m,s} \qquad \forall n,m,s \qquad (s\_trans^{pipe}_{n,m,s}) \qquad (3.2)$$

$$\sum_{p} \mathrm{ex\,p}\_ph_{p,n,m,s} + \sum_{w} wh\_ph_{w,n,m,s} = pipe\_fl_{n,m,s} \qquad \forall n,m,s \qquad \left(tr\_fee_{n,m,s}\right) \qquad (3.3)$$

$$pipe\_fl_{n,m,s} \geq 0, \qquad \forall n,m,s \qquad (3.4)$$

where equation (3.2) is a capacity constraint for an arc nm, (3.3) defines total flow and (3.4) ensures nonnegativity of a decision vector

⬛ *Core model*

🟪 *Investor activity*

🟦 *LTC obligation*

🟧 *Storage activity*

TSO problem: KKT conditions

The KKTs from the corresponding minimization problem are:

$$0 \leq pipe\_fl_{n,m,s} \perp -tr\_fee_{n,m,s} + tr\_costs^{pipe}_{n,m,s} + \ s\_trans^{pipe}_{n,m,s} \geq 0 \qquad \forall n,m,s \quad (3.5)$$

$$0 \leq s\_trans^{pipe}_{n,m,s} \perp pipe\_constr^{pipe}_{n,m,s} + \sum_{s'<s} InvLNG_{n,m,s} - pipe\_fl_{n,m,s} \geq 0 \qquad \forall n,m,s \quad (3.6)$$

$$trans\_fee_{n,m,s} \ \ free, \qquad \sum_{p} \mathrm{ex\,p}\_ph_{p,n,m,s} + \sum_{w} wh\_ph_{w,n,m,s} = pipe\_fl_{n,m,s} \qquad \forall n,m,s \quad (3.7)$$

⬛ *Core model*

🟪 *Investor activity*

🟦 *LTC obligation*

🟧 *Storage activity*

TSO & LNG investor problem: objective function

Investor (the network operator) determines the optimal capacity of gas infrastructure simultaneously with the other players. We assume that he has perfect and complete information about other players. The investor optimizes his discounted net profits from the congestion revenue minus operation and fixed cost. The investment mechanism is such that there will only be a positive investment if the total revenues from congestion fees $s\_trans_{n,m,s}^{pipe}$* for the incremental capacity exceed the total fixed costs $invLNG\_fixc_{n,m,s}^{inv}$**.

$$\max_{InvLNG_{n,m,s},\, pipe\_fl_{n,m,s}} NPV^{tso} = \delta_s \sum_{s} \left\{ \sum\nolimits_{n,m} \left[\left(tr\_fee_{n,m,s} \cdot pipe\_fl_{n,m,s} \right) - tr\_costs_{n,m,s}^{pipe}(pipe\_fl_{n,m,s}) - invLNG\_fixc_{n,m,s}^{inv}\left(InvLNG_{n,m,s} \right) \right] \right\} \tag{4.1}$$

---

* *It can be considered that the congestion price of an arc reflects the marginal willingness to pay of the using parties (i.e. exporters) for an addition unit of capacity.*

** *The assumption of continuous investment is certainly a simplification, but it can be taken in natural gas transmission sector where the increase of compressor capacity allows for increase of pipe/LNG capacity by small amounts.*

TSO & LNG investor problem: constraints

s.t.

$$pipe\_constr^{pipe}_{n,m,s} + \sum_{s'<s} InvLNG_{n,m,s} \geq pipe\_fl_{n,m,s} \qquad \forall n,m,s \qquad (s\_trans^{pipe}_{n,m,s}) \qquad (4.2)$$

$$invLNG\_cap^{inv}_{n,m,s} \geq InvLNG_{n,m,s} \qquad \forall n,m,s \quad (s\_investLNG^{inv}_{n,m,s}) \qquad (x\ 4.3)$$

$$InvLNG_{n,m,s} \geq 0, \qquad \forall n,m,s$$

$$pipe\_fl_{n,m,s} \geq 0, \qquad \forall n,m,s \qquad (4.4)$$

▬ *Core model*

▬ *Investor activity*

▬ *LTC obligation*

▬ *Storage activity*

where equation (4.2) is a capacity constraint for an arc nm, (4.3) defines the investment capacity (4.4) ensures nonnegativity of decision vectors

TSO & LNG investor problem: KKT conditions

The KKTs from the corresponding minimization problem are:

$$0 \leq pipe\_fl_{n,m,s} \perp \delta_s(-tr\_fee_{n,m,s} + tr\_costs^{LNG}_{n,m,s}) + s\_trans^{pipe}_{n,m,s} \geq 0 \qquad \forall n,m,s \qquad (4.5)$$

$$0 \leq InvLNG_{n,m,s} \perp \delta_s \frac{\partial invLNG\_fixc^{inv}_{p,n,s}\left(InvLNG_{n,m,s}\right)}{\partial InvLNG_{n,m,s}} - \sum_{s'<s} s\_trans^{LNG}_{n,m,s} + s\_investLNG_{n,m,s} \geq 0 \qquad \forall n,m,s \qquad (4.6)$$

$$0 \leq s\_trans^{pipe}_{n,m,s} \perp (pipe\_constr^{pipe}_{n,m,s} + \sum_{s'<s} InvLNG_{n,m,s} - pipe\_fl_{n,m,s}) \geq 0 \qquad \forall n,m,s \qquad (4.7)$$

$$0 \leq s\_investLNG_{n,m,s} \perp invLNG\_cap^{inv}_{n,m,s} - InvLNG_{n,m,s} \geq 0 \qquad \forall n,m,s \qquad (x\ 4.8)$$

▬ *Core model*

🟪 *Investor activity*

🟦 *LTC obligation*

🟧 *Storage activity*

(4.6): the investment mechanism is such that there will only be a positive investment if the total revenues from congestion fees $s\_trans^{pipe}_{n,m,s}$ for the incremental capacity exceed the total fixed costs

Storage operator problem (currently not used within MCP formulation)

The storage operator (SO) provides a mechanism to efficiently utilize storage capacity ($cap\_stor_{n,s}$) for the wholesalers (storage capacity virtually is located in *n* wholesaler's node). SO maximizes the profits resulting from withdrawing/selling gas at peak price/demand levels ($sw_{n,s}$) while injecting it at low price/demand levels ($sv_{n,s}$).

$$\max_{sv_{n,s}, sw_{n,s}} \prod_{n}^{stor} = \sum_{s} [p\_b_{n,s} \cdot sw_{n,s} - p\_b_{n,ss} \cdot sv_{n,s}] \quad (5.1)$$

s.t.

$$St_{n,s+1} = St_{n,s} + (1 - loss) \cdot sv_{n,s} - sw_{n,s} \quad \forall n,s \quad (\alpha_{n,s}) \quad (5.2)$$

$$cap\_stor_{n,s}^{st} \geq St_{n,s}, \quad \forall n,s \ (pcst_{n,s}) \quad (5.3)$$

$$inj\_vol_{n,s}^{st} \geq sv_{n,s}, \quad \forall n,s \ (pcv_{n,s}) \quad (5.4)$$

$$with\_vol_{n,s}^{st} \geq sw_{n,s}, \quad \forall n,s \ (pcw_{n,s}) \quad (5.5)$$

Storage operator problem (currently not used within MCP formulation)

The KKTs from the corresponding minimization problem are:

$$\forall n,s: \quad 0 \leq sv_{n,s} \ \perp p\_b_{n,s} + pcv_{n,s} \geq (1 - loss) * \alpha_{n,s} \tag{5.6}$$

$$\forall n,s: \quad 0 \leq sw_{n,s} \ \perp \alpha_{n,s} + pcw_{n,s} \geq p\_b_{n,s} \tag{5.7}$$

$$\forall n,s: \quad 0 \leq St_{n,s} \ \perp \alpha_{n,s+1} + pcst_{n,s} \geq \alpha_{n,s} \tag{5.8}$$

$$\forall n,s: \quad \alpha_{n,s}\ free, \quad St_{n,s+1} = St_{n,s} + (1 - loss) \cdot sv_{n,s} - sw_{n,s} \tag{5.9}$$

$$\forall n,s: \quad 0 \leq pcst_{n,s} \ \perp cap\_stor^{st}_{n,s} \geq St_{n,s} \tag{5.10}$$

$$\forall n,s: \quad 0 \leq pcv_{n,s} \ \perp inj\_vol^{st}_{n,s} \geq sv_{n,s} \tag{5.11}$$

$$\forall n,s: \quad 0 \leq pcw_{n,s} \ \perp with\_vol^{st}_{n,s} \geq sw_{n,s} \tag{5.12}$$

▬ *Core model*

▬ *Investor activity*

▬ *LTC obligation*

▬ *Storage activity*

Market clearing conditions

Market clearing conditions combine the various optimization problem together into the one market-equilibrium problem:

$$\sum_{p}\sum_{m}(\exp\_sale_{p,m,n,s}) = \sum_{w}\sum_{m}(wh\_sale_{w,n,m,s}) + sw_{n,s} - sv_{n,s} \qquad \forall n,s \quad (p\_b_n \quad free) \tag{6.1}$$

$$\sum_{p}\exp\_ph_{p,n,m,s} + \sum_{w} wh\_ph_{w,n,m,s} = pipe\_fl_{n,m,s} \qquad \forall n,m,s \quad (trans\_fee_{n,m,s} \;\; free) \tag{6.2}$$

$$p\_FC_{n,s} - \left(a_{n,s} + b_{n,s}\cdot\sum_{w}\sum_{m\neq n} wh\_sale_{w,m,n,s}\right) = 0 \qquad \forall n,s \;\; (p\_FC_{n,s} free) \tag{6.3}$$

Where:

(6.1) is satisfied if the entire quantity of gas imported by each downstream player equals to the entire gas quantity consumed, transported, or injected/withdrawed into storage.

(6.2) is used to define the total physical gas flow which also serves as a market clearing condition between TSO and exporters/traders

(6.3) guarantees that the price for final consumers matches the inverse demand function at the equilibrium point

Representation of perfectly competitive & Cournot players

The parameter $\theta_{\mathrm{m}} \in [0,1]$ is used to define different model settings of either Cournot competition or perfect competition is assumed. In the case of perfect competition ($\theta_{\mathrm{m}} = 0$) each player is a price-taker (players “perceives” price as a parameter – the conjecture that a change in own supply will not induce a change of a market price). A value of ($\theta_{\mathrm{m}} = 1$) means the Cournot player.

$$\max_{wh\_sale_{w,n,m,s},\, wh\_ph_{w,n,m,s}} \Pi^{ws}_{w,n}$$

$$= \sum_{s} \left\{ \sum_{m} \left(wh\_sale_{w,n,m,s} \cdot p\_FC_{m,s}(\cdot) \right) - \sum_{mm} \left(wh\_sale_{w,m,n,s} \cdot p\_b_{n,s} \right) - \sum_{n \to m \in N} tr\_fee_{n,m,s} \cdot wh\_ph_{w,n,m,s} \right\}$$

$$0 \leq wh\_sale_{w,n,m,s}$$

$$\perp -p\_FC_{m,s} + \theta_{\mathrm{m}} \frac{\partial p\_FC_{m,s}\left(wh\_sale_{w,n,m,s}\right)}{\partial wh\_sale_{w,n,m,s}} \cdot wh\_sale_{w,n,m,s} + p\_b_{m,s} + s\_whbal^{whs}_{w,n,s} - s\_whbal^{whs}_{w,m,s} \geq 0$$

Where $\frac{\partial p\_FC_{m,s}\left(wh\_sale_{w,n,m,s}\right)}{\partial wh\_sale_{w,n,m,s}}$ *is a slope of demand function (*$b_{m,s}$*).*

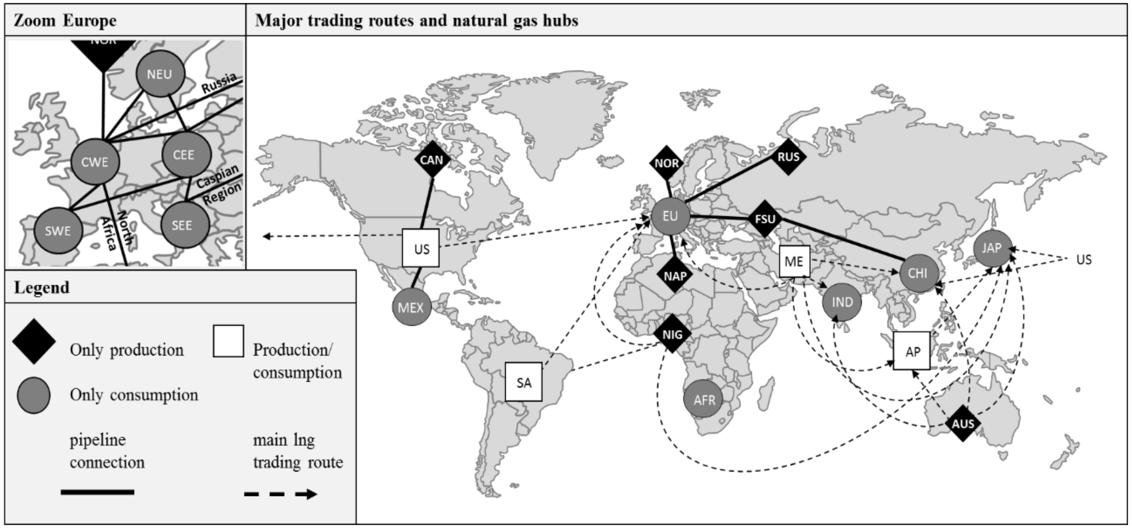
Solving the MCP model

- ✓ The combination of all the Karush-Kuhn-Tucker conditions and the market clearing conditions form the MCP.
- ✓ All profit functions are concave and differentiable, all cost functions are convex and differentiable, thus, the KKT points are necessary and sufficient for the optimal solution.
- ✓ Numerical problems in MCP format can be efficiently solved with PATH* solver.
- ✓ Model is formulated and solved in GAMS** software.

** The PATH algorithm relies on the key ideas of Newton's method for solving a system of nonlinear equations, namely: generation of a sequence of approximate solutions by solving linear approximations to the nonlinear equations; constructing each linear approximation in the sequence as a first-order Taylor approximation about the previous approximate solutions; and, when necessary to speed convergence, adjusting the generated sequence of approximate solutions by damping procedure.*

*** The General Algebraic Modelling System (GAMS) is a modeling system used for mathematical programming and optimization.*

Example of MCP model implementation



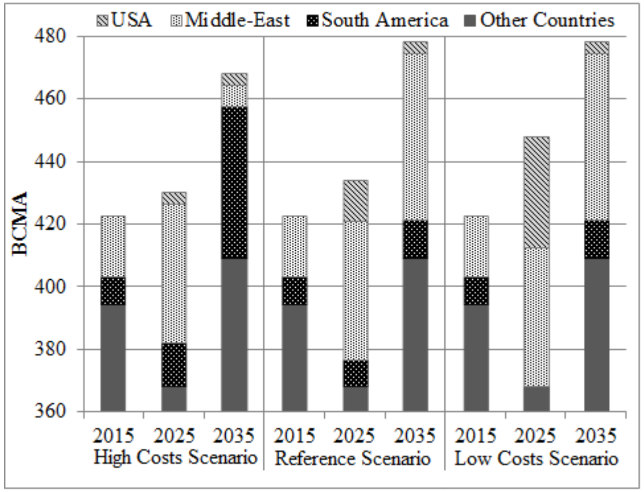
Geographical coverage of the model

Key questions analyzed:

- To which extent different world regions can be affected by US LNG terminals' capacity expansion and corresponding export volumes?
- How other gas suppliers will react to a higher, or lower, US export volumes overseas?
- How flexible US investment decisions with regard to LNG capacity costs?

Montenegro, R., Riepin, I., Hauser P. (2016): **"Modelling of world LNG market development: focus on US investments and supplies"**, IEEE Conference Proceedings EEM 2016, DOI: 10.1109/EEM.2016.7521361

Supplier's share from Europe's import



Content:

NLP formulation: objective function

The **objective function maximizes social welfare** (or minimize loss in social welfare) over the whole time intervals. SW is obtained by adding consumers' and producers' surplus at the equilibrium point. Therefore, we calculate it as an area formed by the inverse demand and supply functions. This quantity represents the net gain of all participants in the market. **Advantage:** social welfare optimization problem is relatively *easy* to solve (as long as it doesn't incorporate integer or stochastic elements).

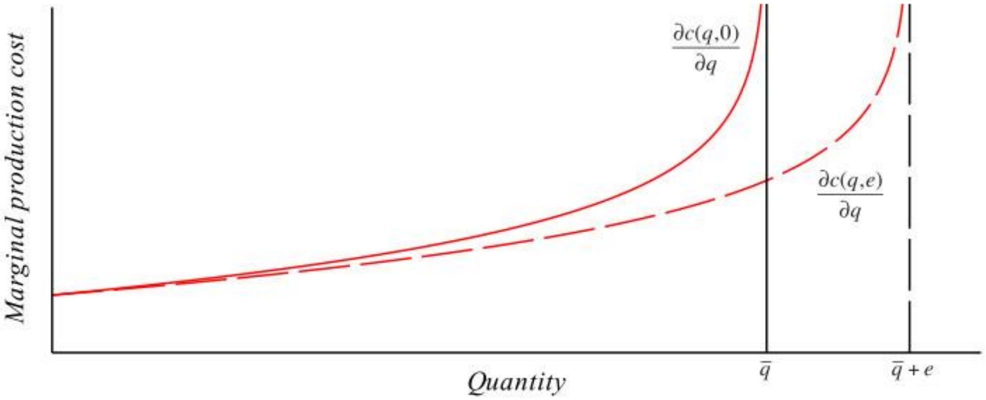
$$\max SW = \sum_{s} \left\{ \begin{array}{ll} 0{,}5 \cdot \sum\limits_{w,n,m\neq n} \left[\left(a_{m,s} - p\_FC_{m,s} \right) \cdot wh\_sale_{w,n,m,s} \right] & \text{…consumer surplus} \\ + \sum\limits_{w,n,m\neq n} \left(wh\_sale_{w,n,m,s} \cdot p\_FC_{m,s} \right) & \text{…producer surplus} \\ - \sum\limits_{p,n,m\neq n} \left(exp\_sale_{p,m,n,s} \cdot mpc_{n,t}^{prod} \right) & \text{…production cost} \\ - \sum\limits_{n,m\neq n} \left(pipe\_fl_{n,m,s} \cdot tr\_costs_{n,m,s}^{pipe} \right) & \text{…transport cost} \\ - \left(\sum\limits_{m} sv_{m,s} \cdot cost_{m}^{store.in} + sv_{m,s} \cdot cost_{m}^{store.out} \right) & \text{…storage cost} \end{array} \right\}$$

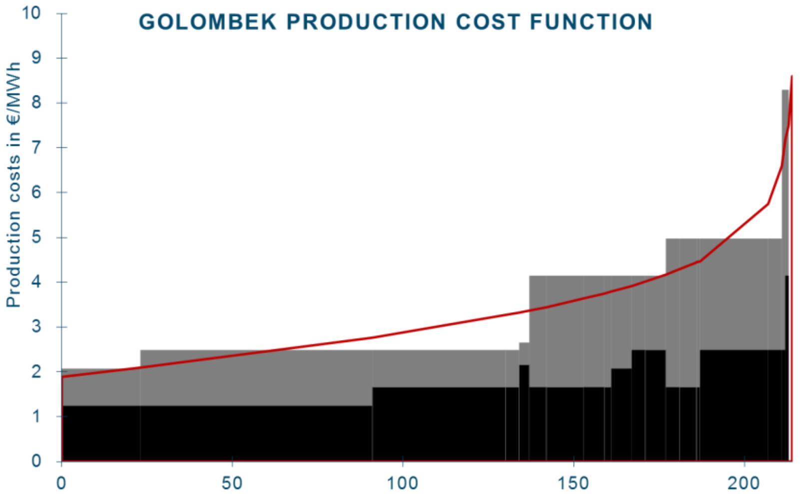
NLP formulation: production cost function

NLP formulation allowed to incorporate production cost function proposed by Golombek et al. (1995). The marginal supply cost curve is expressed as follows:

$$mpc(q) = \alpha + \beta \cdot q + \gamma \cdot ln(\frac{Cap - q}{Cap})$$

Where $\alpha > 0$ is the minimum marginal unit cost term; $\beta \geq 0$ is the per unit linearly-increasing cost term; and $\gamma \leq 0$ is the term that induces high marginal cost when production is close to full capacity. The parameters for the production cost function were originally derived to fit merit-order type production costs function, and afterwards updated based on available open sources.





Solving the NLP model

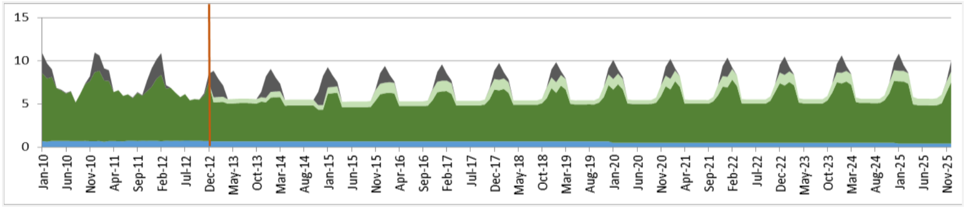
- ✓ The NLP problem is formed by maximizing the SW objective function under the set of constraints that are identical to the MCP model.
- ✓ Nonlinearities occur due to inverse demand function and Golombek production cost function (both terms are convex*).
- ✓ After some workarounds IPOPT** solver was chosen for the resulting NLP model.

** Huppmann, Daniel, Endogenous Investment Decisions in Natural Gas Equilibrium Models with Logarithmic Cost Functions (November 1, 2012). DIW Berlin Discussion Paper No. 1253.*

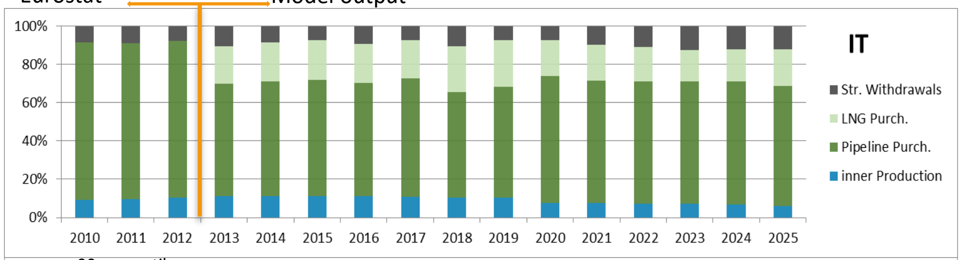
** *https://www.coin-or.org/Ipopt/documentation/node3.html*

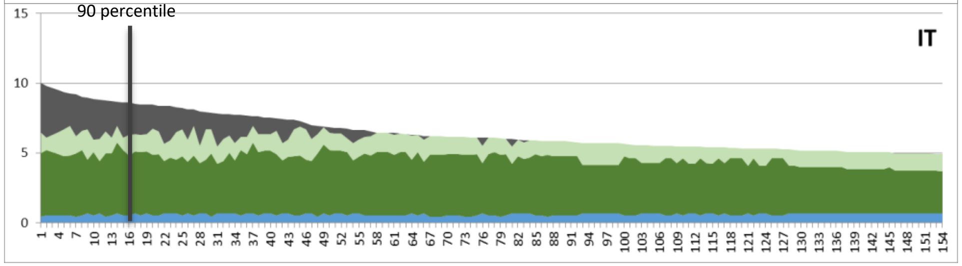
Example of NLP model implementation

Gas demand fluctuations



Eurostat Model output

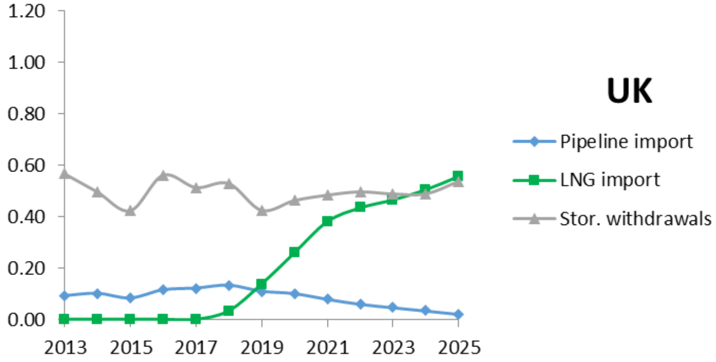




Load duration curve

“Natural Gas Storages in Competition with Alternative Flexibility Sources” presented on the COLEDD conference, Sep 2016.

Which supply source brings most flexibility to meet demand fluctuations?



Coefficient of variation yearly for selected countries

The objective of the work was to analyze the future role of storages and their position in competition with other flexibility sources to meet European countries’ specific demand fluctuations.

Content:

1. Definitions and input data proceeding
2. MCP model: formulation and applications
3. NLP model: formulation and applications
4. Discussion

MCP vs NLP model

$$\min F(x)$$

$$s.t. \quad g_i(x) \leq 0 \quad (\lambda_i) \quad \forall i = 1, \ldots n$$

$$h_i(x) = 0 \quad (\mu_i) \quad \forall j = 1, \ldots m$$

$$\nabla f(x) + \sum_{i=1}^{n} \lambda_i \nabla \mathrm{g}_i(\mathrm{x}) + \sum_{j=1}^{n} \mu_i \nabla \mathrm{h}_j(\mathrm{x}) = 0$$

$$0 \geq g_i(x) \perp \lambda_i \geq 0 \quad \forall i = 1, \ldots n$$

$$0 = h_i(x) \quad \mu_j \text{ free} \quad \forall j = 1, \ldots m$$

Mixed complementarity problem (MCP)

KKT 1

…

KKT n

Subject to:

Constraints *(capacity, balances, clearing conditions)*

$$\min \sum_{J=1}^{n} f_j(x_j)$$

$$s.t.$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i$$

$$0 \leq x_j \leq u_j$$

$$(i = 1 \ldots . m); \; (j = 1 \ldots . n)$$

Nonlinear problem (NLP)

Objective function (maximization of social welfare)

Subject to:

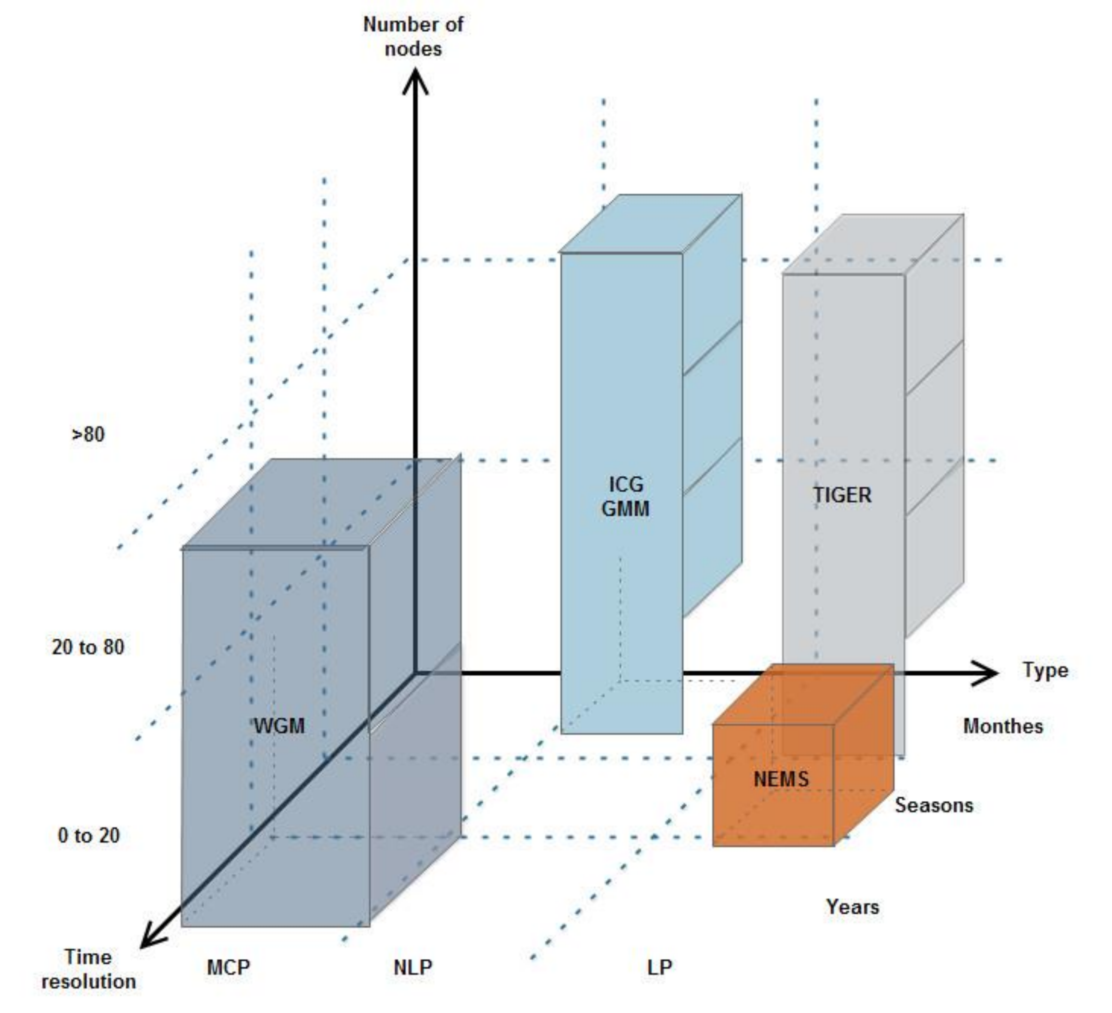
Constraints *(capacity, balances, clearing conditions)*

Gas market models

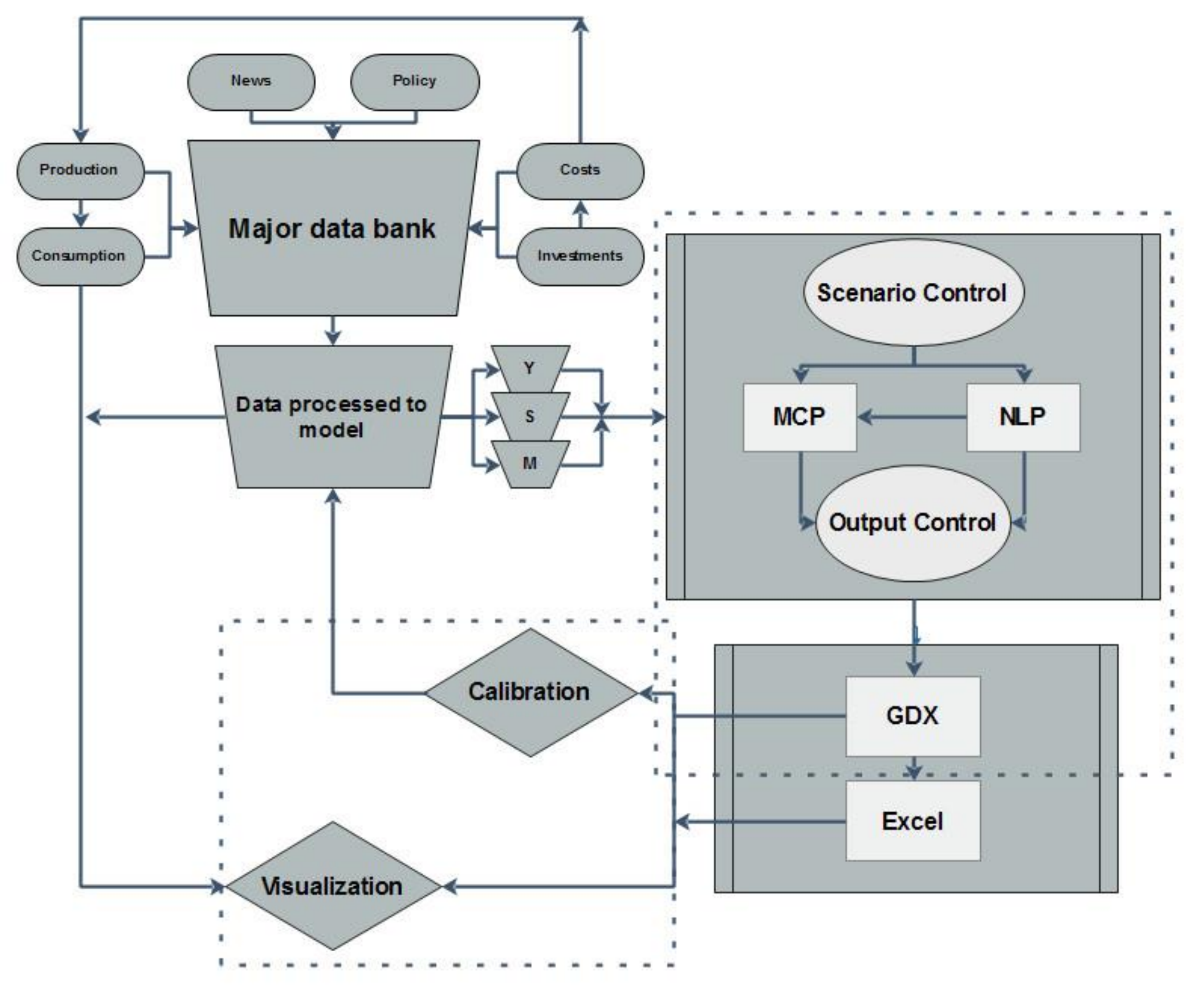
Name	Author	Type	Region	MP	Nodes	Time Scale	Time resolution	Seasons	Dynamics
NEMS	EIA U.S.	LP	North America	no	15	2030	1 year	2	yes
ICF GMM	ICF Int.	NLP	US	no	114	several years	monthly	12	no
WGM	Egging	MCP	World	yes	41	2030	5 years	2	yes
FRISBEE	Statistics Norway	PE	World	no	13	2030	1 year	1	yes
COLUMBUS	Hecking and Panke, EWI	MCP	World	yes	-	2050	monthly(?)	12(2)	yes
GASMOD	Holz, DIW Berlin	MCP	Europe	yes	6	2025	10 years	1	yes
GASTALE	Lise and Hobbs	MCP	Europe	yes	19	2030	5 years	3	yes
TIGER	Lochner et al., EWI	LP	Europe	no	-	2020	monthly	12	yes
NATGAS	Zwart and Mulder	MCP	Europe	yes	-	2035	5 years	2	yes
-	**BTU LSEW**	**MCP** **NLP**	**Europe** **World**	**yes**	**> 80**	**2030**	**Monthly** **Quarterly** **Yearly**	**12**	**yes**

Thank you!

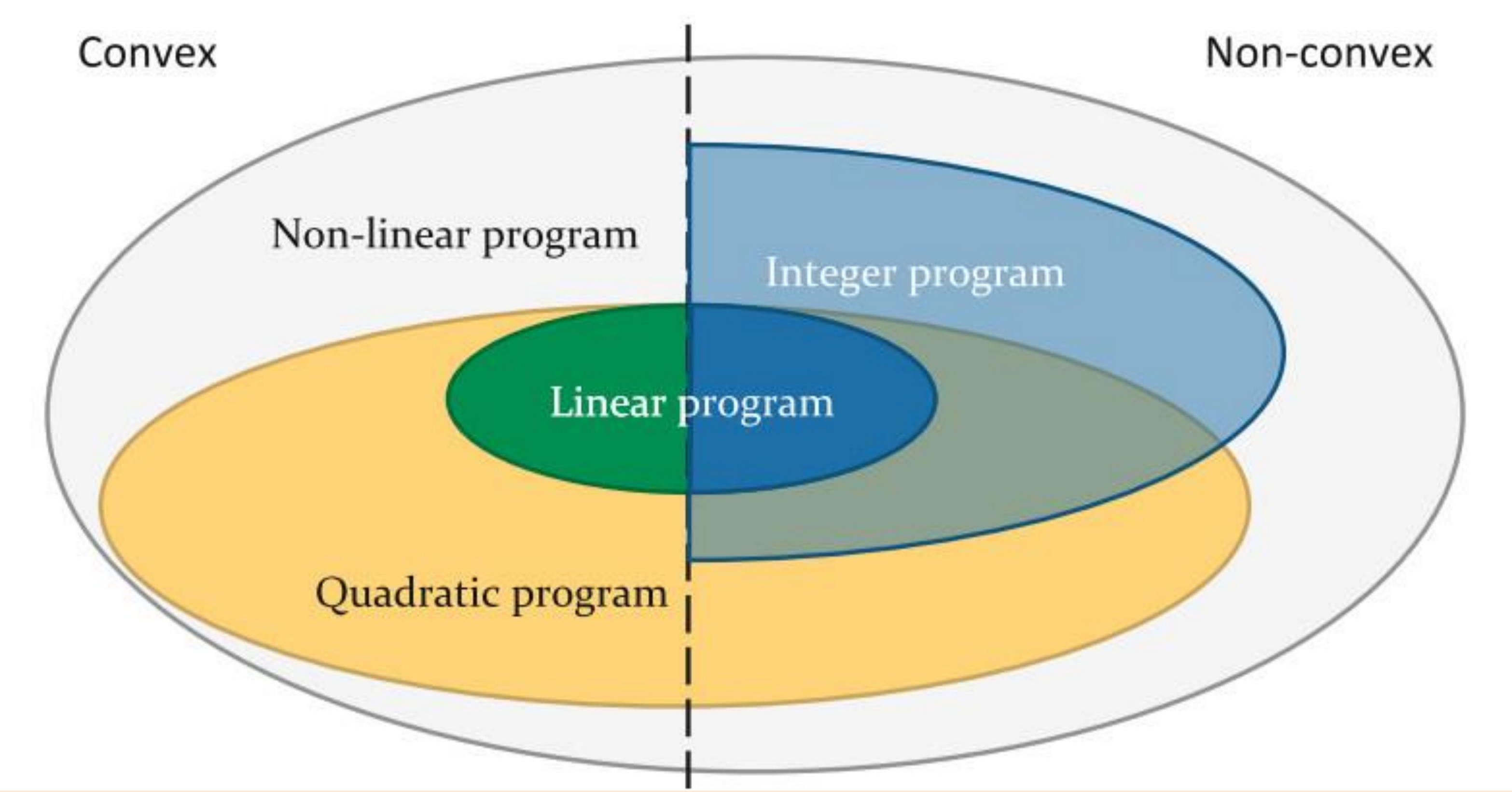
Gas market models



Model architecture: schematic overview



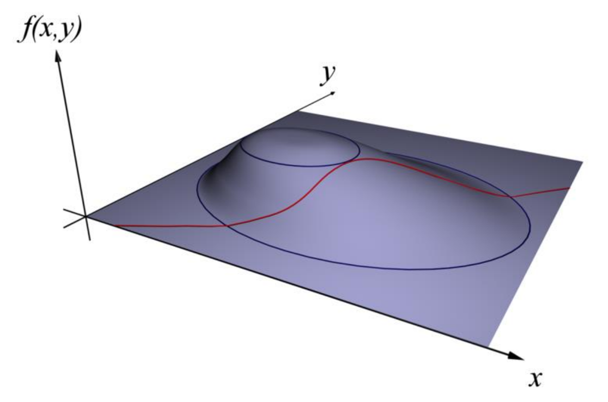
Types of optimization problems



InfraTrain 2014 – One and Two-level Energy Market Equilibrium Modelling,
Daniel Huppmann

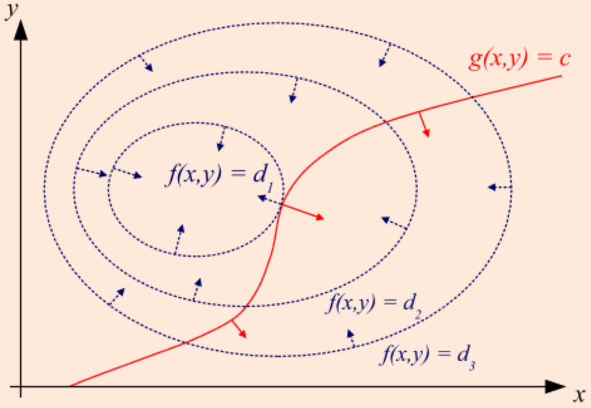
MCP / Method of Lagrange multipliers: problem definition

In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*



$$max\, f(x,y)$$

$$s.t.\, g(x,y) = c$$



Where f(x,y) – objective function
g(x,y) - constraint

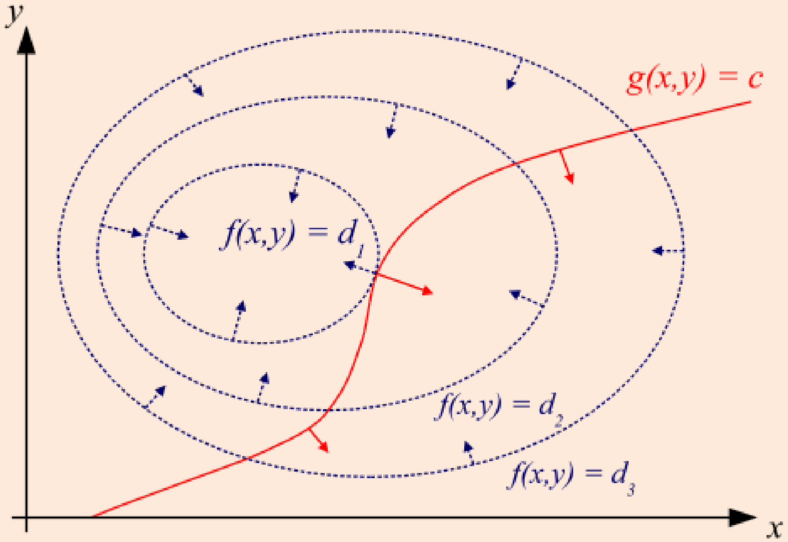
MCP / Method of Lagrange multipliers

In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*

$$\max f(x, y)$$

$$s.t.\, g(x, y) = c$$

Key point: 2 curves are tangent at the same point -> i.e. they have the same slope



MCP / Method of Lagrange multipliers

In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*

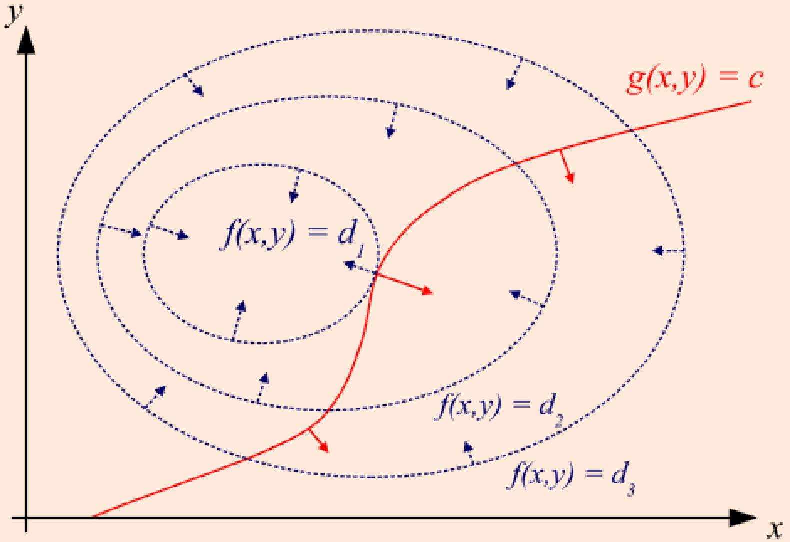
$$\max f(x,y)$$

$$s.t.\ g(x,y) = c$$

$$\nabla f(x,y) = \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$$

$$\nabla g(x,y) = \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}$$

Gradient of the function shows the direction of the max increase of the function:



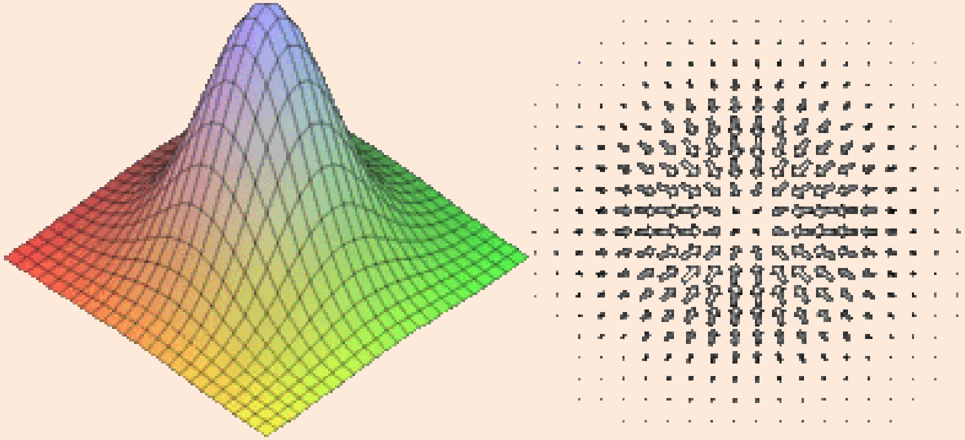
MCP / Method of Lagrange multipliers: Gradient

In mathematics, the gradient is a *generalization of the usual concept of derivative of a function in one dimension to a function in several dimensions.*

- ✓ Gradient points in the direction of the greatest rate of increase of the function and its magnitude is the slope of the graph in that direction

$$\nabla f = \frac{\partial f}{\partial x_1}\mathbf{e}_1 + \cdots + \frac{\partial f}{\partial x_n}\mathbf{e}_n$$

where the e_i are the orthogonal unit vectors pointing in the coordinate directions.



MCP / Method of Lagrange multipliers

In mathematical optimization, the method of Lagrange multipliers *is a strategy for finding the local maxima and minima of a function subject to equality constraints:*

$$max\, f(x, y)$$

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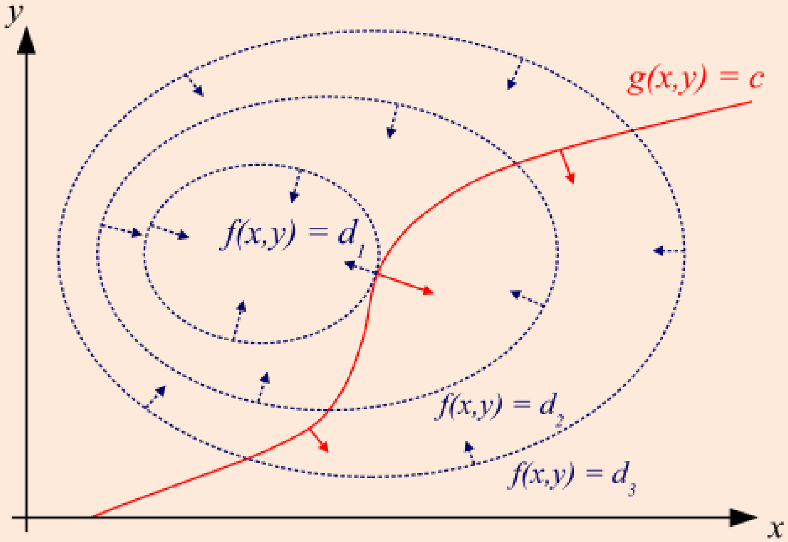
If 2 vectors are orthogonal to the same slope, it has to be the case that they are parallel:

$$\nabla f(x, y) = \lambda \cdot \nabla g(x, y)$$



MCP / Method of Lagrange multipliers: economical interpretation

- ✓ In economics the optimal profit to a player is calculated subject to a constrained space of actions, where a Lagrange multiplier *is the change in the optimal value of the objective function (profit) due to the relaxation of a given constraint*

$$\nabla f(x,y) = \lambda \cdot \nabla g(x,y)$$

$$\Downarrow$$

$$\frac{\partial L(x,y)}{\partial g(x,y)} = \lambda$$

in such a context λ is the marginal cost of the constraint, and is referred as the shadow price