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The costs of ignoring uncertainty: an illustrative toy model

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Toy model

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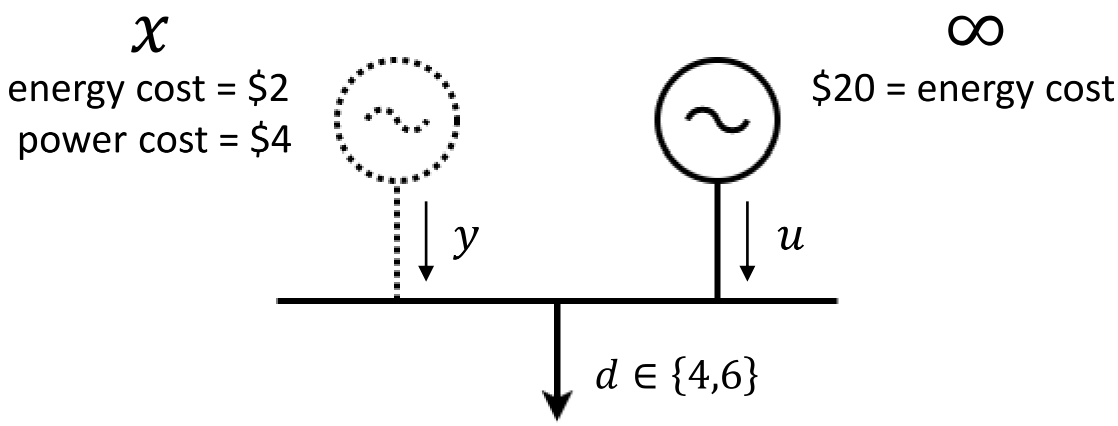

- Consider an insular power system whose energy demand is supplied by a generating unit at the cost of **$20 per energy unit**. This source is expensive but has **unlimited capacity**.
- The future energy **demand is uncertain**, but it may take solely two values, either **4 or 6 energy units**.
- The system planner consider building a generating unit. The operating cost of this unit is **$2 per energy unit**, and its investment cost is **$4 per power unit**.

A two-stage stochastic program

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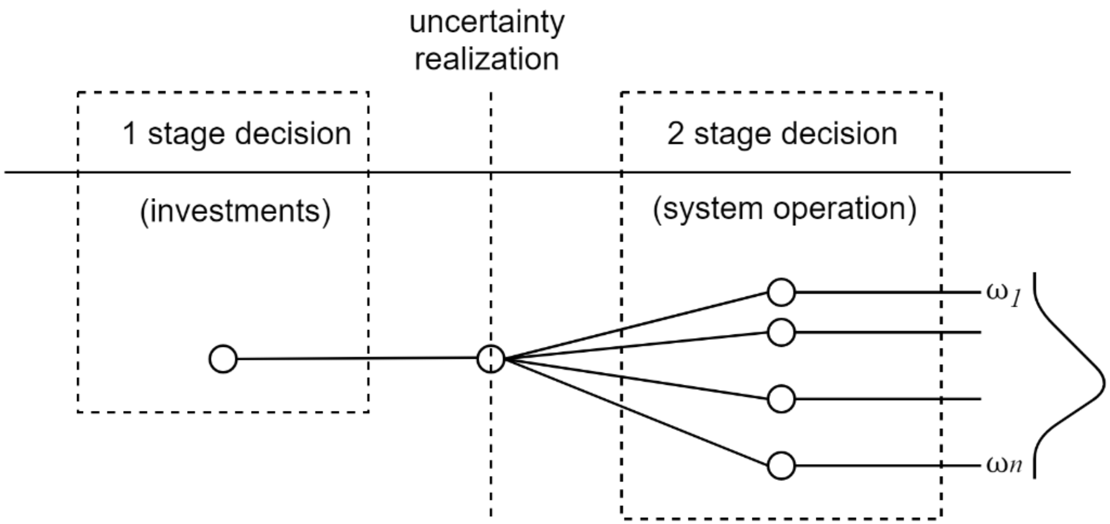


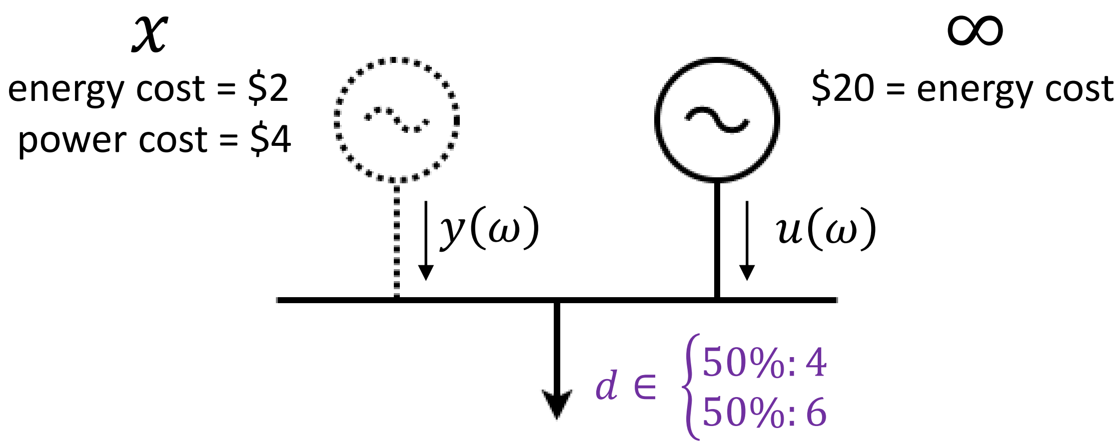
The system planner needs to make an investment decision under uncertainty. A "classical" two-stage stochastic program can be formulated as follows:

$$\min_{x \in X} \varphi(x, \omega) = c^T x + \mathbb{E}[Q(y(x, \omega))]$$

Where:

x	the vector of first-stage decisions
ω	the vector of uncertain outcomes
$y(x, \omega)$	the vector of second-stage decisions

Toy model



The standard approach to solve this problem numerically:

i. Assume that vector ω has a finite number of realizations (scenarios) $\omega_1 .. \omega_n$ with respective (positive) probabilities $p_1 .. p_n | \sum_1^n p = 1$

ii. Then a two-stage stochastic problem can be reformulated with a deterministic LP equivalent

$$\min_{x, y_1, \ldots, y_n} c^T x + \sum_{n=1}^{N} p_n \; Q(y_n(x, \omega), u_n(x, \omega))$$

Toy model: a stochastic problem and its numerical solution

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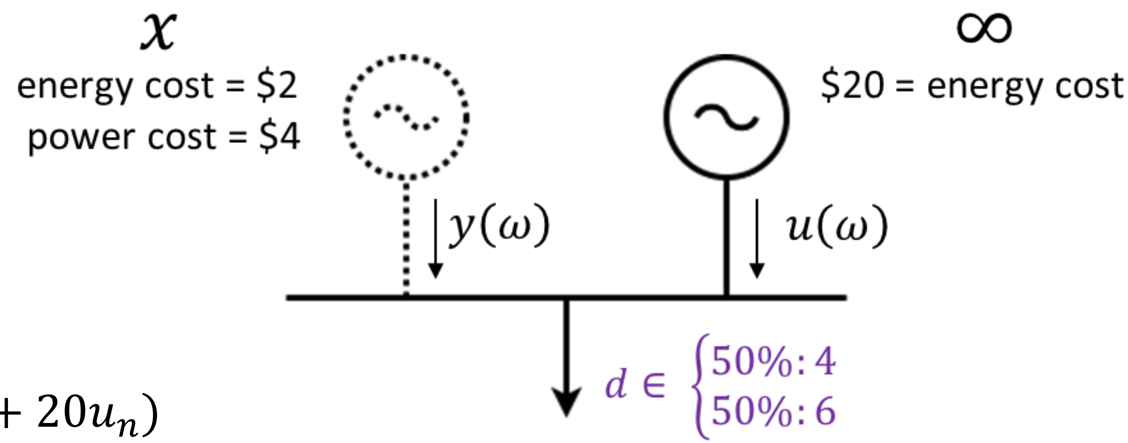
Notation:

SP	stochastic problem
SS	stochastic solution
x, y_n, u_n	decision variables



$$SP: SS = \min_{x, y_n, u_n \geq 0} 4x + \sum_{n=1}^{2} \frac{1}{2}(2y_n + 20u_n)$$

$$\begin{aligned} s.t. \quad & y_1 + u_1 = 4\ (\omega_1) \\ & y_2 + u_2 = 6\ (\omega_2) \\ & y_n \leq x\ \forall n \end{aligned}$$

⇨

$$\boldsymbol{x = 6}$$

$$\boldsymbol{SS} = 4 \cdot 6 + \frac{1}{2}(2 \cdot 4) + \frac{1}{2}(2 \cdot 6) = \boldsymbol{\$34}$$

Toy model: an expected value problem and its numerical solution

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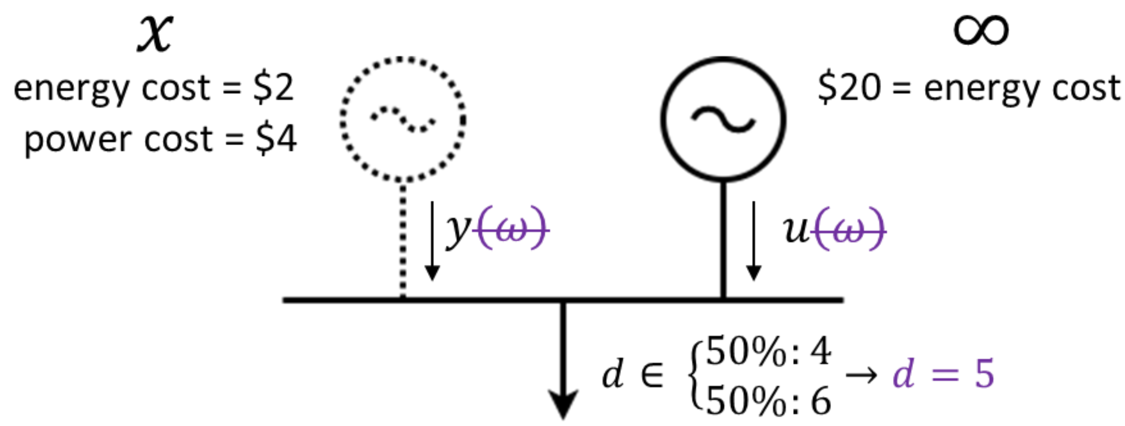
Notation:

EVP	expected value problem
EV	solution of EVP problem
x, y, u	decision variables



$$EVP: EV = \min_{x,y,u \geq 0} 4x + 2y + 20u$$

$$s.t. \quad y + u = 5$$
$$y \leq x$$

⇒

$x = 5$

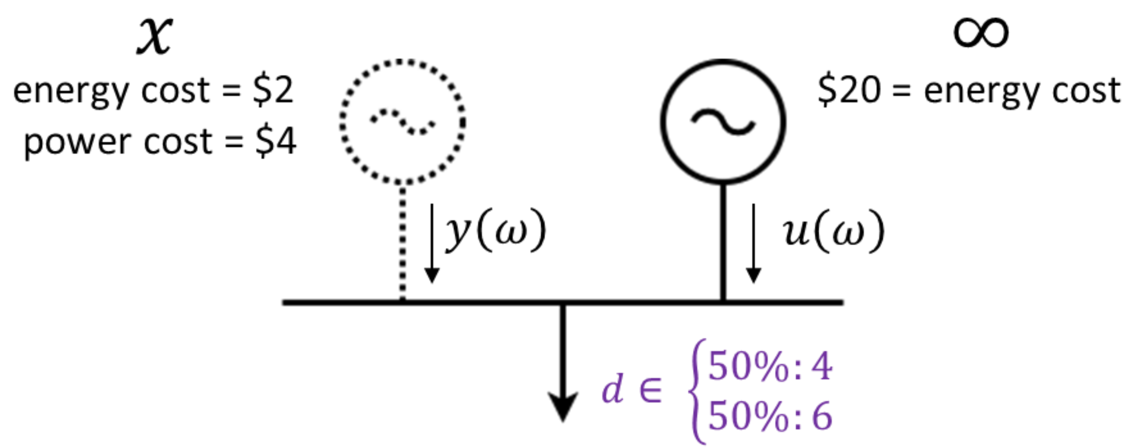
$EV = 4 \cdot 5 + 2 \cdot 5 = \$30$

4 + 2*6 =

Toy model: evaluating the expected costs of the naïve solution

Notation:

EEV	expected costs of the (naïve) solution of expected value problem
y_n, u_n	decision variables



$$EEV = \min_{x=5, y_n, u_n} 4 \cdot 5 + \sum_{n=1}^{2} \frac{1}{2}(2y_n + 20u_n)$$

$$\text{s.t.} \quad \begin{aligned} y_1 + u_1 &= 4 \ (\omega_1) \\ y_2 + u_2 &= 6 \ (\omega_2) \\ y_n &\leq 5 \ \forall n \end{aligned}$$

$\Rightarrow$

$$\boldsymbol{EEV} = 4 \cdot 5 + \frac{1}{2}(2 \cdot 4) + \frac{1}{2}(2 \cdot 5 + 20 \cdot 1) = \mathbf{\$39}$$

The Expected Costs of Ignoring Uncertainty (ECIU)

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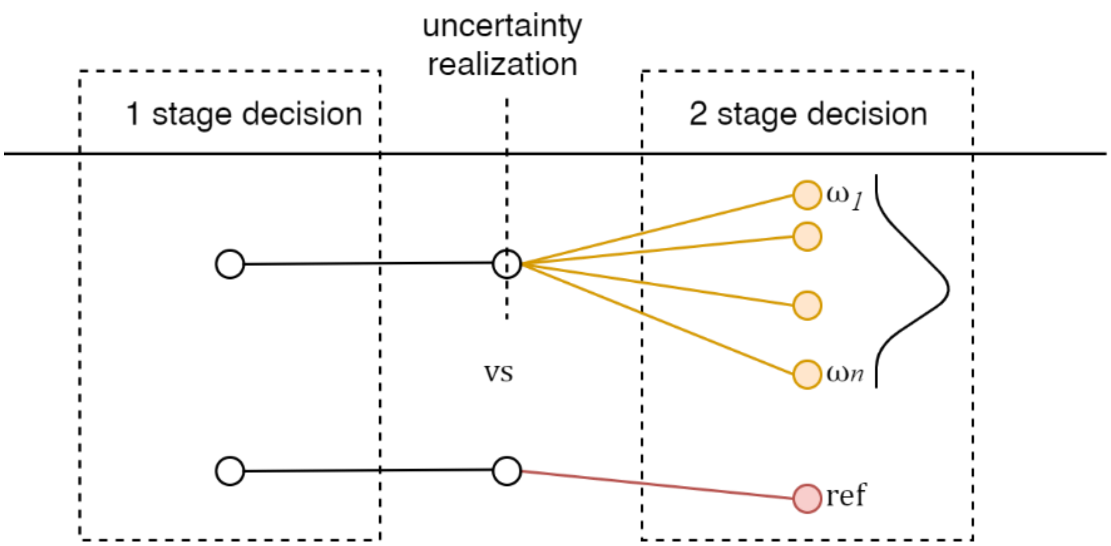

When comparing the two approaches (ignoring uncertainty versus modeling uncertainty explicitly) the natural question to ask is **how much difference it really makes to the quality of the decisions reached?**

The ECIU measures the value of using a stochastic model (or the expected costs of ignoring uncertainty when using a deterministic model).

$$\boldsymbol{ECIU = EEV - SS = \$39 - \$34 = \$5}$$

Toy model: the added value of perfect information

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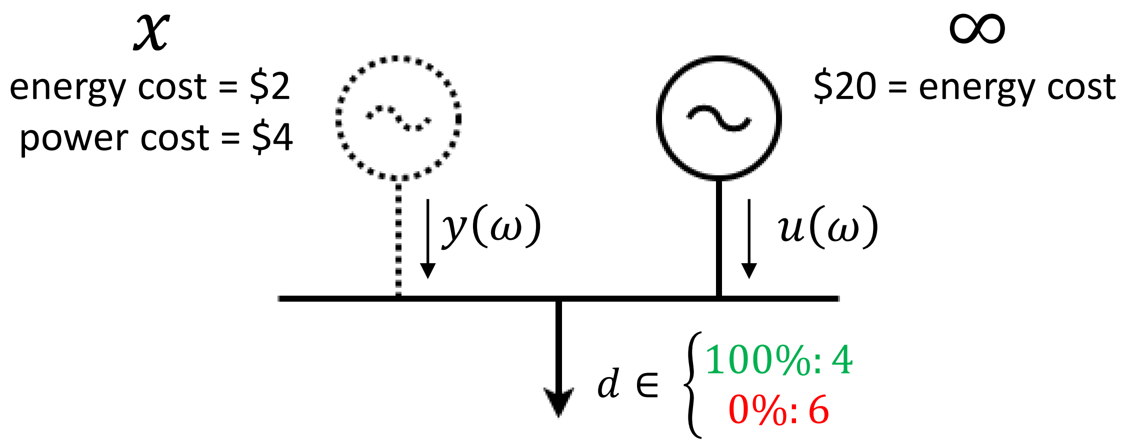

If system planner **knew** at the first stage **which scenario will play out**, it could **optimize** an expansion plan (i.e. that results in lower cost) for that scenario.

The expected value (and the corresponding mathematical problem) of such solution is denoted in the literature as **„wait-and-see"** solution (or wait-and-see (WS) problem).

The difference between the (probability-weighted) **wait-and-see** solutions and the **here-and-now (stochastic)** solution represents the **added value of information about the future** (i.e., the expected profit).

Toy model: the added value of perfect information, numerical solution

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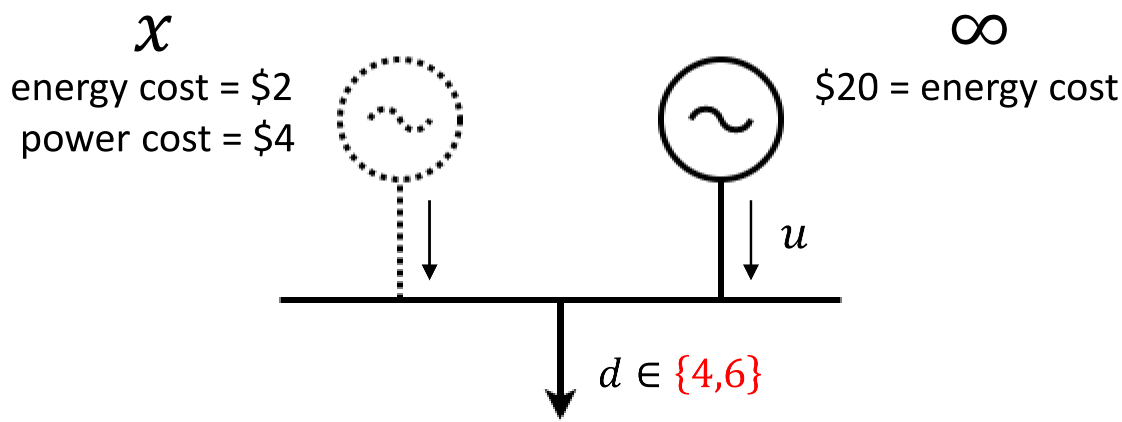
Notation:

WS	wait-and-see solution (assuming that planner has perfect information)



$$WS1 = \min_{x,y,u\geq 0} 4x + 2y + 20u$$

$$y + u = 4$$
$$y \leq x$$

$$\boldsymbol{x = 4}$$
$$WS1 = 4 \cdot 4 + 2 \cdot 4 = \$\mathbf{24}$$

$$WS2 = \min_{x,y,u\geq 0} 4x + 2y + 20u$$

$$y + u = 6$$
$$y \leq x$$

$$\boldsymbol{x = 6}$$
$$WS2 = 4 \cdot 6 + 2 \cdot 6 = \$\mathbf{36}$$

The Expected Value of Perfect Information (EVPI)

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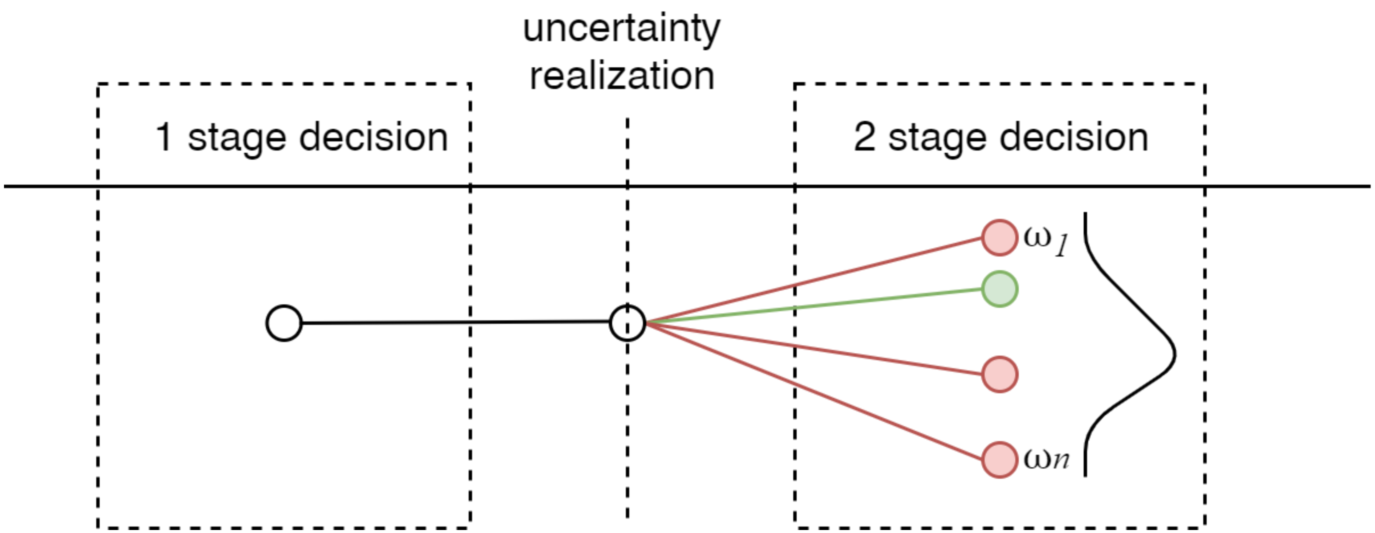


[model perspective] How much the expected costs could be reduced if system planner in the first stage knew exactly which scenario would happen?

[economic perspective] An upper bound to the amount that should be paid for improved forecasts.

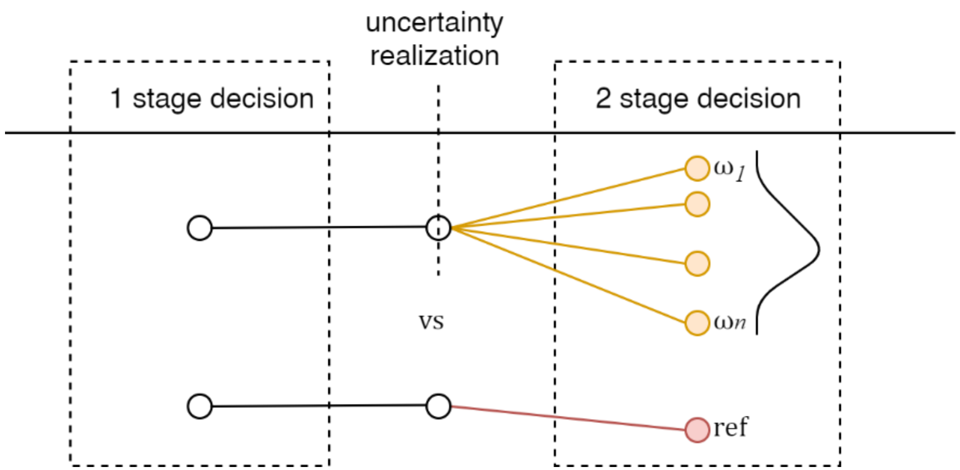
$$\boldsymbol{EVPI = SS - \sum_{n=1}^{N} p_n \cdot WS_n = \$34 - \frac{1}{2}(\$24 + \$36) = \$4}$$

Both ECIU and EVPI compare the expected value of the (investment) decision with another decision made without uncertainty.

ECIU: an investment decision is made when uncertainty is ignored.

⇩

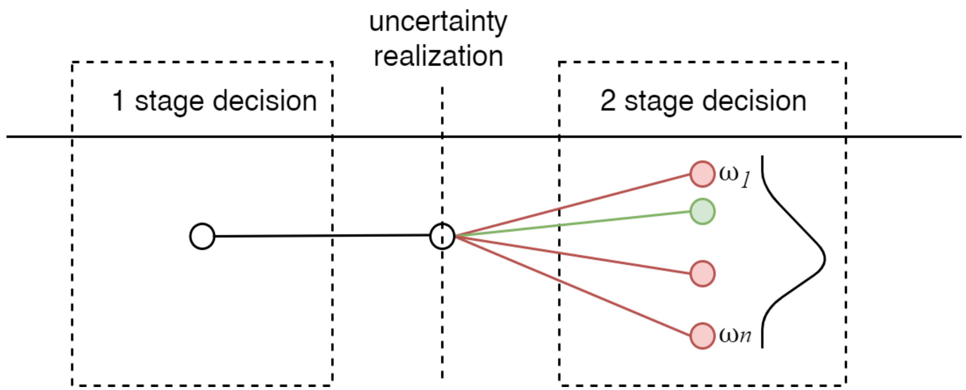
The ECIU is the additional expected cost of assuming that future is certain



EVPI: an investment decision is made after uncertainty is removed.

⇩

The EVPI is the expected cost of being uncertain about the future



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