Brandenburgische
Technische Universität
Cottbus - Senftenberg

Robust optimization of coordinated electricity and gas system expansion

30 October 2019

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Multistage stochastic programming

The “classical” two-stage stochastic program can be formulated as follows:

$$\min_{x \in X} \varphi(x, \omega) = \ c^T x + \mathbb{E}[Q(y(x, \omega))]$$

Where:

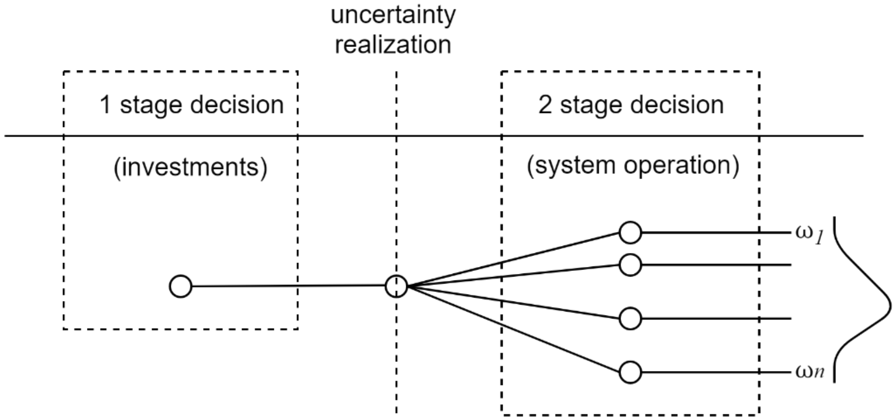
x	represent the vector of first-stage decisions
ω	represent the vector of uncertain outcomes
$y(x, \omega)$	represent the vector of second-stage decisions

Multistage stochastic programming

The standard approach to solve this problem numerically:

i. Assume that vector ω has a finite number of realizations (scenarios) $\omega_1 .. \omega_n$

ii. with respective (positive) probabilities $p_1 .. p_n | \sum_1^n p = 1$



iii. Then the problem can be reformulated with a deterministic LP equivalent

$$\min_{x,y_1,\ldots,y_n} \; c^T x + \sum_{n=1}^{N} p_n \; Q(y_n(x,\omega))$$

Adaptive robust optimization

$$\min_{x \in X} \max_{u \in U} \min_{y \in Y(x,u)} f(x, y, u)$$

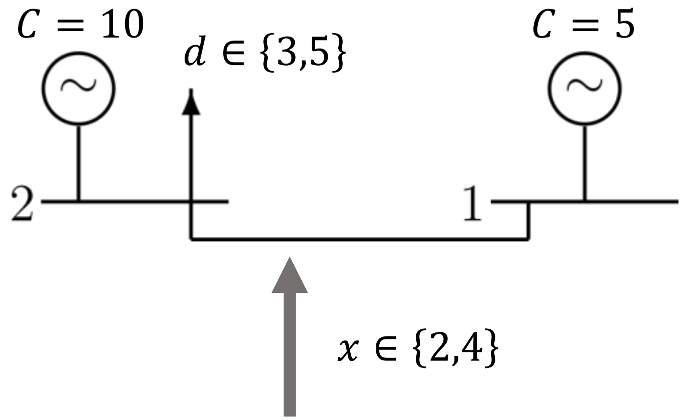
Adaptive robust optimization

Source: Antonio J. Conejo

A lecture on adaptive robust optimization, DTU,

13 June 2019.

https://www.youtube.com/watch?v=Zk6y8joQLNQ



- A planner has the option of building just one of two alternative lines with a capacity of 2 or 4. Per unit cost of investment is 1.
- At node 1, a generator with per unit production cost of 5.
- At node 2, a generator with per unit production cost of 10.
- Demand is at node 2 and can take values either 3 or 5.

Adaptive robust optimization

Source: Antonio J. Conejo

A lecture on adaptive robust optimization, DTU,

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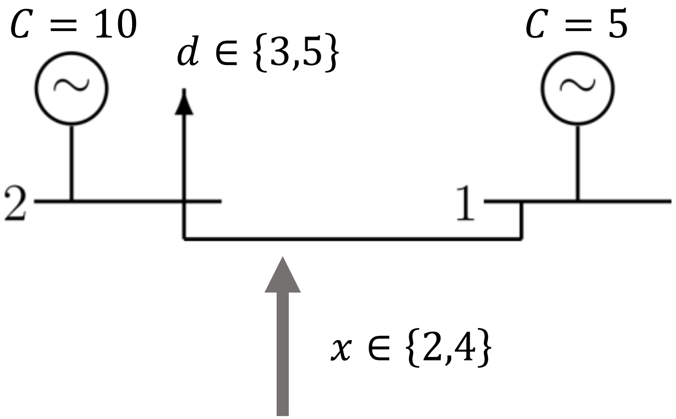


$$\min_{x\in\{2,4\}} \max_{d\in\{3,5\}} \min_{y_1,y_2\geq 0} Z = 1x + 5\,y_1 + 10y_2$$

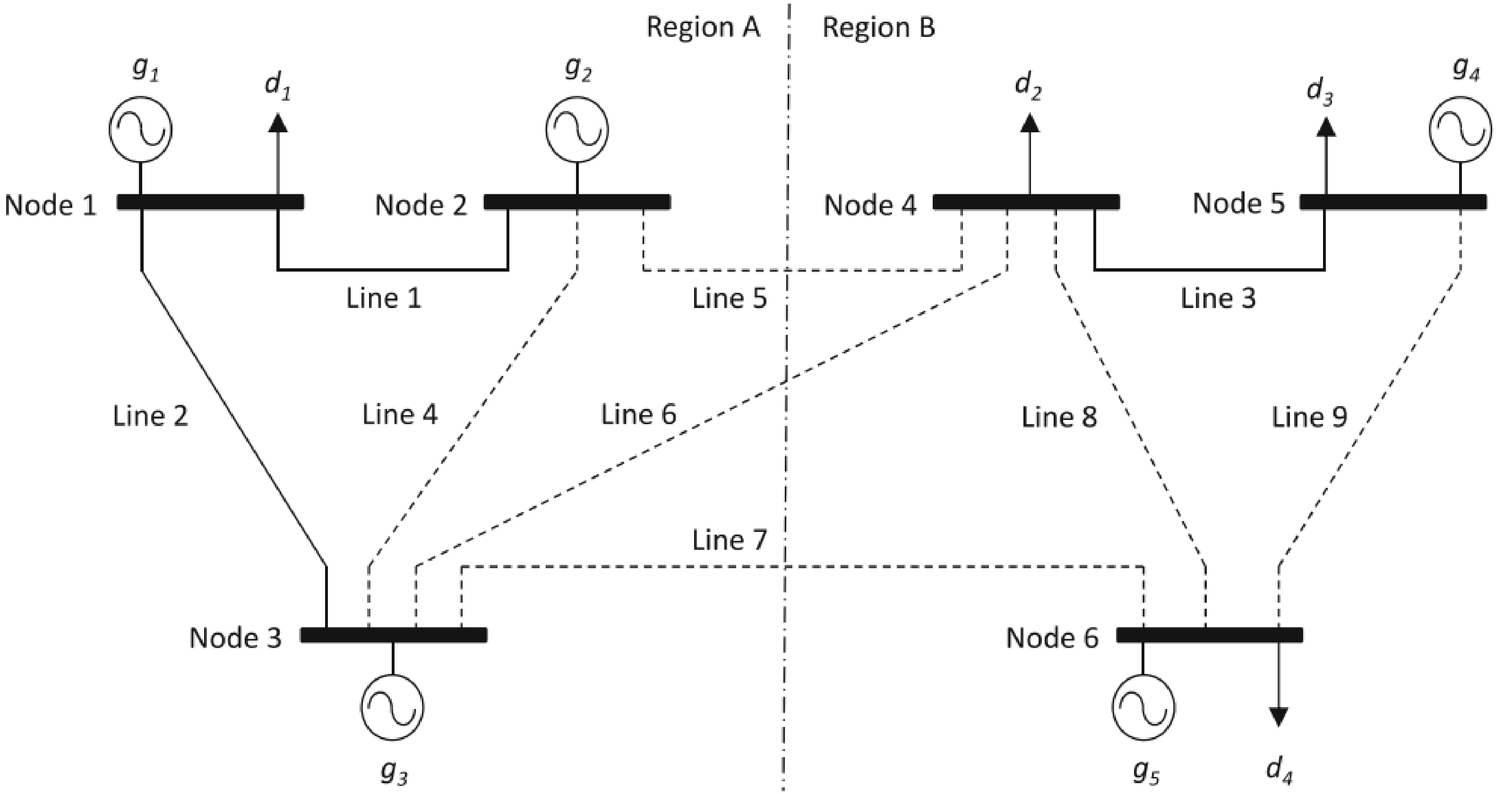
$$s.t. \qquad y_1 + y_2 = 10$$

$$y_1 \leq x$$

Given the simplicity of this problem, it can be solved by enumeration.

... a few important notes before moving on.

Illustrative case: 6-node system



An illustrative case is based on Conejo, A. J., Baringo, M. L., Kazempour, S. J., & Siddiqui, A. S. (2016). *Investment in Electricity Generation and Transmission: Decision Making under Uncertainty.*

Deterministic MINLP

$$\min_{\Delta} \sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_{g} C_g^E P_g^E + \sum_{d} C_d^{LS} P_d^{LS} \right] \tag*{[1.a]}$$

s.t.

$$\Delta = \{X_l^L, P_l^L, P_g^E, P_d^{LS}, \theta_n\}$$

$$\sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^L \le I^{\widetilde{max}} \tag*{[1.b]}$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \tag*{[1.c]}$$

$$\sum_{g\in\Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d\in\Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \tag*{[1.d]}$$

$$P_l^L = B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \setminus l \in \Omega^{L+} \tag*{[1.e]}$$

$$P_l^L = X_l^L B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \in \Omega^{L+} \tag*{[1.f]}$$

$$-F_l^{max} \le P_l^L \le F_l^{max} \quad \forall l \tag*{[1.g]}$$

$$0 \le P_g^E \le P_d^{Emax} \quad \forall g \tag*{[1.h]}$$

$$0 \le P_d^{LS} \le P_d^{Dmax} \quad \forall d \tag*{[1.i]}$$

$$-\pi \le \theta_n \le \pi \quad \forall n \tag*{[1.j]}$$

$$\theta_n = 0 \quad n{:}\, ref. \tag*{[1.k]}$$

Deterministic MINLP

$$\min_{\Delta} \sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_{g} C_g^E P_g^E + \sum_{d} C_d^{LS} P_d^{LS} \right] \qquad \text{[1.a]}$$

s.t.

$$\Delta = \{X_l^L, P_l^L, P_g^E, P_d^{LS}, \theta_n\}$$

$$\sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^L \le I^{\widetilde{max}} \qquad \text{[1.b]}$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \qquad \text{[1.c]}$$

$$\sum_{g\in\Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d\in\Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \qquad \text{[1.d]}$$

$$P_l^L = B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \setminus l \in \Omega^{L+} \qquad \text{[1.e]}$$

nonlinear term

$$P_l^L = X_l^L B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \in \Omega^{L+} \qquad \text{[1.f]}$$

$$-F_l^{max} \le P_l^L \le F_l^{max} \quad \forall l \qquad \text{[1.g]}$$

$$0 \le P_g^E \le P_d^{Emax} \quad \forall g \qquad \text{[1.h]}$$

$$0 \le P_d^{LS} \le P_d^{Dmax} \quad \forall d \qquad \text{[1.i]}$$

$$-\pi \le \theta_n \le \pi \quad \forall n \qquad \text{[1.j]}$$

$$\theta_n = 0 \quad n{:}\,ref. \qquad \text{[1.k]}$$

Deterministic MILP

$$\min_{\Delta} \sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^L + \sigma \left[\sum_{g} C_g^E P_g^E + \sum_{d} C_d^{LS} P_d^{LS} \right] \qquad \text{[1.a]}$$

s.t.

$$\Delta = \{X_l^L, P_l^L, P_g^E, P_d^{LS}, \theta_n\}$$

$$\sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^L \le \widetilde{I^{max}} \qquad \text{[2.b]}$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \qquad \text{[2.c]}$$

$$\sum_{g\in\Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d\in\Omega_n^D} \left(P_d^{Dmax} - P_d^{LS}\right) \quad \forall n \qquad \text{[2.d]}$$

$$P_l^L = B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \setminus l \in \Omega^{L+} \qquad \text{[2.e]}$$

$$-F_l^{max} \le P_l^L \le F_l^{max} \quad \forall l \setminus l \in \Omega^{L+} \qquad \text{[2.f]}$$

MILP reformulation

$$-X_l^L F_l^{max} \le P_l^L \le X_l^L F_l^{max} \quad \forall l \in \Omega^{L+} \qquad \text{[2.g]}$$

$$-(1 - X_l^L)M \le P_l^L - B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \le (1 - X_l^L)M \quad \forall l \in \Omega^{L+} \qquad \text{[2.h]}$$

$$0 \le P_g^E \le P_d^{Emax} \quad \forall g \qquad \text{[2.i]}$$

$$0 \le P_d^{LS} \le P_d^{Dmax} \quad \forall d \qquad \text{[2.j]}$$

$$-\pi \le \theta_n \le \pi \quad \forall n \qquad \text{[2.k]}$$

$$\theta_n = 0 \quad n{:}\,ref. \qquad \text{[2.l]}$$

Uncertainty sets

Introducing polyhedral uncertainty set and “uncertainty budget” constraints:

$$P_g^{Emax} \in \left[0, \overline{P_g^{Emax}}\right] \qquad \forall g$$

$$\frac{\sum_g(\overline{P_g^{Emax}} - P_g^{Emax})}{\sum_g(\overline{P_g^{Emax}})} \leq \Gamma^G$$

$if\ \Gamma^G = 0 \rightarrow P_g^{Emax} = \overline{P_g^{Emax}}$

$if\ \Gamma^G = 1 \rightarrow P_g^{Emax} \in \left[0, \overline{P_g^{Emax}}\right]$

$if\ \Gamma^G = 0.2$ → up to 20% of generation capacity may be unavailable

$$P_d^{Dmax} \in \left[\underline{P_d^{Dmax}}, \overline{P_d^{Dmax}}\right] \quad \forall d$$

$$\frac{\sum_d(P_d^{Dmax} - \underline{P_d^{Dmin}})}{\sum_g(\overline{P_d^{Dmax}} - \underline{P_d^{Dmax}})} \leq \Gamma^D$$

$$\Gamma^G, \Gamma^D = \{0..1\}$$

Creating a monster

Let’s merge together:

1. Adaptive robust problem
2. MILP formulation of expansion planning problem
3. Uncertainty sets

$$\min_{X_l^L}\ \max_{P_g^{Emax},P_d^{Dmax}}\ \min_{P_g^E,P_d^{LS},P_l^L,\theta_n}\ \sum_{l\in\Omega^{L+}} \widetilde{I}_l X_l^L + \sigma\left[\sum_g C_g^E P_g^E + \sum_d C_d^{LS} P_d^{LS}\right]$$

$$\sum_{g\in\Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d\in\Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \tag*{[3.b]}$$

$$P_l^L = B_l\big(\theta_{s(l)} - \theta_{r(l)}\big) \quad \forall l \setminus l \in \Omega^{L+} \tag*{[3.c]}$$

$$P_l^L = X_l^L B_l\big(\theta_{s(l)} - \theta_{r(l)}\big) \quad \forall l \in \Omega^{L+} \tag*{[3.d]}$$

$$-F_l^{max} \le P_l^L \le F_l^{max} \quad \forall l \tag*{[3.e]}$$

$$0 \le P_g^E \le P_d^{Emax} \quad \forall g \tag*{[3.f]}$$

$$0 \le P_d^{LS} \le P_d^{Dmax} \quad \forall d \tag*{[3.g]}$$

$$-\pi \le \theta_n \le \pi \quad \forall n \tag*{[3.h]}$$

$$\theta_n = 0 \quad n{:}\ ref. \tag*{[3.i]}$$

$$P_g^{Emax} \in \left[0, \overline{P_g^{Emax}}\right] \quad \forall g \tag*{[3.j]}$$

$$\frac{\sum_g\left(\overline{P_g^{Emax}} - P_g^{Emax}\right)}{\sum_g\left(\overline{P_g^{Emax}}\right)} \le \Gamma^G \tag*{[3.k]}$$

$$P_d^{Dmax} \in \left[\underline{P_d^{Dmax}}, \overline{P_d^{Dmax}}\right] \quad \forall d \tag*{[3.l]}$$

$$\frac{\sum_d\left(P_d^{Dmax} - \underline{P_d^{Dmin}}\right)}{\sum_g\left(\overline{P_d^{Dmax}} - \underline{P_d^{Dmax}}\right)} \le \Gamma^D \tag*{[3.m]}$$

$$\sum_{l\in\Omega^{L+}} \widetilde{I}_l X_l^L \le I^{\widehat{max}} \tag*{[3.n]}$$

$$X_l^L = \{0,1\} \quad \forall l \in \Omega^{L+} \tag*{[3.o]}$$

Merging problems of level 2 and level 3

Karush–Kuhn–Tucker conditions

- Let us consider the problem:

$$\min_x F(x) \qquad [4.a]$$

$$s.t. \quad g_i(x) \leq 0 \quad (\lambda_i) \qquad \forall i = 1, \dots n \qquad [4.b]$$

$$h_j(x) = 0 \quad (\mu_j) \qquad \forall j = 1, \dots m \qquad [4.c]$$

- For this problem, the KKT conditions are:

$$\nabla f(x) + \sum_{i=1}^{n} \lambda_i \nabla g_i(x) + \sum_{j=1}^{n} \mu_j \nabla h_j(x) \leq 0 \perp x \geq 0 \qquad [4.d]$$

$$0 \geq g_i(x) \perp \lambda_i \geq 0 \qquad \forall i = 1, \dots n \qquad [4.e]$$

$$0 = h_j(x) \qquad \mu_j \text{ free} \qquad \forall j = 1, \dots m \qquad [4.f]$$

The solution stationarity is ensured by the equation [4.d].
Equations [4.e] and [4.f] ensure complementarity and feasibility of a solution

Merging problems of level 2 and level 3

$$\underset{\Delta^{SUB}}{max}\ \sigma\left[\sum_{g} C_g^E P_g^E + \sum_{d} C_d^{LS} P_d^{LS}\right]$$

$$\Delta^M = \{P_d^{Emax}, P_d^{Dmax}, P_g^E, P_d^{LS}, P_l^L, \theta_n,$$
$$\lambda_n, \phi_l^L, \phi_l^{L+}, \phi_l^{Lmax}, \phi_l^{Lmin}, \phi_l^{Emax}, \phi_l^{Emin}, P_d^{Dmax}, \phi_l^{Dmin}, \phi_l^{Nmax}, \phi_l^{Nmin},$$

$$\mathrm{P_g^{Emax}} \in \left[0, \overline{\mathrm{P_g^{Emax}}}\right] \quad \forall \mathrm{g} \qquad [5.ab]$$

$$\frac{\sum_g(\overline{P_g^{Emax}} - P_g^{Emax})}{\sum_g(\overline{P_g^{Emax}})} \leq \Gamma^G \qquad [5.ac]$$

$$P_d^{Dmax} \in \left[\underline{P_d^{Dmax}}, \overline{P_d^{Dmax}}\right] \quad \forall d \qquad [5.ad]$$

$$\frac{\sum_d(P_d^{Dmax} - \underline{P_d^{Dmin}})}{\sum_g(\overline{P_d^{Dmax}} - \underline{P_d^{Dmax}})} \leq \Gamma^D \qquad [5.ae]$$

$$\sum_{g\in\Omega_n^E} P_g^E - \sum_{l|s(l)=n} P_l^L + \sum_{l|r(l)=n} P_l^L = \sum_{d\in\Omega_n^D} (P_d^{Dmax} - P_d^{LS}) \quad \forall n \qquad [5.af]$$

$$P_l^L = B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \setminus l \in \Omega^{L+} \qquad [5.ag]$$

$$P_l^L = X_l^{L*} B_l\left(\theta_{s(l)} - \theta_{r(l)}\right) \quad \forall l \in \Omega^{L+} \qquad [5.ah]$$

$$-F_l^{max} \leq P_l^L \leq F_l^{max} \quad \forall l \qquad [5.ai]$$

$$0 \leq P_g^E \leq P_d^{Emax} \quad \forall g \qquad [5.aj]$$

$$0 \leq P_d^{LS} \leq P_d^{Dmax} \quad \forall d \qquad [5.ak]$$

$$-\pi \leq \theta_n \leq \pi \quad \forall n \qquad [5.al]$$

$$\theta_n = 0 \quad n{:}\,ref. \qquad [5.am]$$

… … [tbc]

For KKT we need:

1. Feasibility conditions
    1. Constraints of level 2
    2. Constraints of level 3
2. Stationarity conditions
3. Complementarity conditions

Merging problems of level 2 and level 3

…… [tbc]

$$\sigma C_g^E - \lambda_{n(g)} + \phi_g^{Emax} - \phi_g^{Emin} = 0 \quad \forall g \qquad [5.an]$$

$$\sigma C_d^{LS} - \lambda_{n(d)} + \phi_d^{Dmax} - \phi_d^{Dmin} = 0 \quad \forall d \qquad [5.ao]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^L + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \setminus l \in \Omega^{L+} \qquad [5.ap]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^{L+} + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \in \Omega^{L+} \qquad [5.aq]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} = 0 \quad \forall n \setminus n{:}\,ref \qquad [5.ar]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} - \chi^{ref} = 0 \quad n{:}\,ref \qquad [5.as]$$

$$0 \le \phi_l^{Lmax} \perp F_l^{max} - P_l^L \ge 0 \quad \forall l \qquad [5.at]$$

$$0 \le \phi_l^{Lmin} \perp P_l^L + F_l^{max} \ge 0 \quad \forall l \qquad [5.au]$$

$$0 \le \phi_l^{Emax} \perp P_d^{Emax} - P_g^E \ge 0 \quad \forall g \qquad [5.av]$$

$$0 \le \phi_l^{Emin} \perp P_g^E \ge 0 \quad \forall g \qquad [5.aw]$$

$$0 \le \phi_l^{Dmax} \perp P_d^{Dmax} - P_d^{LS} \ge 0 \quad \forall d \qquad [5.ax]$$

$$0 \le \phi_l^{Dmin} \perp P_d^{LS} \ge 0 \quad \forall d \qquad [5.ay]$$

$$0 \le \phi_l^{Nmax} \perp \pi - \theta_n \ge 0 \quad \forall n \qquad [5.az]$$

$$0 \le \phi_l^{Nmin} \perp \theta_n + \pi \ge 0 \quad \forall n \qquad [5.ba]$$

For KKT we need:

1. Feasibility conditions
    1. Constraints of level 2
    2. Constraints of level 3
2. Stationarity conditions
3. Complementarity conditions

Merging problems of level 2 and level 3

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…… [tbc]

$$\sigma C_g^E - \lambda_{n(g)} + \phi_g^{Emax} - \phi_g^{Emin} = 0 \quad \forall g \qquad [5.an]$$

$$\sigma C_d^{LS} - \lambda_{n(d)} + \phi_d^{Dmax} - \phi_d^{Dmin} = 0 \quad \forall d \qquad [5.ao]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^L + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \setminus l \in \Omega^{L+} \qquad [5.ap]$$

$$\lambda_{s(l)} - \lambda_{r(l)} - \phi_l^{L+} + \phi_l^{Lmax} - \phi_l^{Lmin} = 0 \quad \forall l \in \Omega^{L+} \qquad [5.aq]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} = 0 \quad \forall n \setminus n{:}\,ref \qquad [5.ar]$$

$$\sum_{l \setminus l \in \Omega^{L+} | s(l)=n} B_l \phi_l^L + \sum_{l \in \Omega^{L+} | s(l)=n} X_l^{L*} B_l \phi_l^{L+} - \sum_{l \setminus l \in \Omega^{L+} | r(l)=n} B_l \phi_l^L - \sum_{l \in \Omega^{L+} | r(l)=n} X_l^{L*} B_l \phi_l^{L+} + \phi_l^{Nmax} - \phi_l^{Nmin} - \chi^{ref} = 0 \quad n{:}\,ref \qquad [5.as]$$

For KKT we need:

1. Feasibility conditions
    1. Constraints of level 2
    2. Constraints of level 3
2. Stationarity conditions
3. Complementarity conditions

again nonlinear terms…
but there is a trick!

$$\left\{ \begin{array}{lll}
0 \leq \phi_l^{Lmax} \perp F_l^{max} - P_l^L \geq 0 & \forall l & [5.at] \\
0 \leq \phi_l^{Lmin} \perp P_l^L + F_l^{max} \geq 0 & \forall l & [5.au] \\
0 \leq \phi_l^{Emax} \perp P_d^{Emax} - P_g^E \geq 0 & \forall g & [5.av] \\
0 \leq \phi_l^{Emin} \perp P_g^E \geq 0 & \forall g & [5.aw] \\
0 \leq \phi_l^{Dmax} \perp P_d^{Dmax} - P_d^{LS} \geq 0 & \forall d & [5.ax] \\
0 \leq \phi_l^{Dmin} \perp P_d^{LS} \geq 0 & \forall d & [5.ay] \\
0 \leq \phi_l^{Nmax} \perp \pi - \theta_n \geq 0 & \forall n & [5.az] \\
0 \leq \phi_l^{Nmin} \perp \theta_n + \pi \geq 0 & \forall n & [5.ba]
\end{array} \right.$$

MIP reformulation of complementarity terms

$$0 \leq a \perp b \geq 0$$

Implies:

$$a \geq 0, b \geq 0 \; and \; ab = 0$$

It can be reformulated as:

$$a \geq 0$$

$$b \geq 0$$

$$a \leq Mu$$

$$b \leq M(1-u)$$

Where:

- u is an auxiliary binary variable
- M is a large positive constant

Column-and-constraint generation (or Berders-primal) algorithm

Zeng, B., and Long Z.“Solving two-stage robust optimization problems using a column-and-constraint generation method.” *Oper. Res. Lett.* 41 (2013): 457-461.

Bertsimas, D. et al. “Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem.” *IEEE Transactions on Power Systems* 28 (2013): 52-63.

[A book] Conejo, A. J., Baringo, M. L., Kazempour, S. J., & Siddiqui, A. S. (2016). *Investment in Electricity Generation and Transmission: Decision Making under Uncertainty.* DOI: 10.1007/978-3-319-29501-5

Master problem

$$\min_{\Delta^M} \sum_{l\in\Omega^{L+}} \widetilde{I}_l X_l^L + \eta$$

$$\Delta^M = \{X_l^L, P_{l,v^i}^L, P_{g,v^i}^E, P_{d,v^i}^{LS}, \theta_{n,v^i}, \eta\}$$

$$\sum_{l\in\Omega^{L+}} \widetilde{I}_l X_l^L \leq \widetilde{I^{max}} \quad \text{[6.b]}$$

$$X_l^L = \{0{,}1\} \quad \forall l \in \Omega^{L+} \quad \text{[6.c]}$$

$$\sum_{g\in\Omega_n^E} P_{g,v^i}^E - \sum_{l|s(l)=n} P_{l,v^i}^L + \sum_{l|r(l)=n} P_{l,v^i}^L = \sum_{d\in\Omega_n^D} \left(P_{d,v^i}^{Dmax*} - P_{d,v^i}^{LS}\right) \quad \forall n, \forall v^i \leq v \quad \text{[6.d]}$$

$$P_{l,v^i}^L = B_l \left(\theta_{s(l),v^i} - \theta_{r(l),v^i}\right) \quad \forall l \setminus l \in \Omega^{L+}, \forall v^i \leq v \quad \text{[6.e]}$$

$$P_{l,v^i}^L = X_l^L B_l \left(\theta_{s(l),v^i} - \theta_{r(l),v^i}\right) \quad \forall l \in \Omega^{L+}, \forall v^i \leq v \quad \text{[6.f]}$$

$$-F_l^{max} \leq P_{l,v^i}^L \leq F_l^{max} \quad \forall l, \forall v^i \leq v \quad \text{[6.g]}$$

$$0 \leq P_{g,v^i}^E \leq P_{d,v^i}^{Emax*} \quad \forall g, \forall v^i \leq v \quad \text{[6.i]}$$

$$0 \leq P_{d,v^i}^{LS} \leq P_{d,v^i}^{Dmax*} \quad \forall d, \forall v^i \leq v \quad \text{[6.i]}$$

$$-\pi \leq \theta_{n,v^i} \leq \pi \quad \forall n, \forall v^i \leq v \quad \text{[6.j]}$$

$$\theta_{n,v^i} = 0 \quad n{:}\, ref. \forall v^i \leq v \quad \text{[6.k]}$$

$$\eta \geq \sigma \left\lfloor \sum_g C_g^E P_{g,v^i}^E + \sum_d C_d^{LS} P_{d,v^i}^{LS} \right\rfloor \quad \forall v^i \leq v \quad \text{[6.l]}$$

Three things to consider:

1. Auxiliary variable η
2. Constraints [6.d] – [6.l] have index v
3. P_{d,v^i}^{Dmax*} and P_{d,v^i}^{Emax*} are fixed

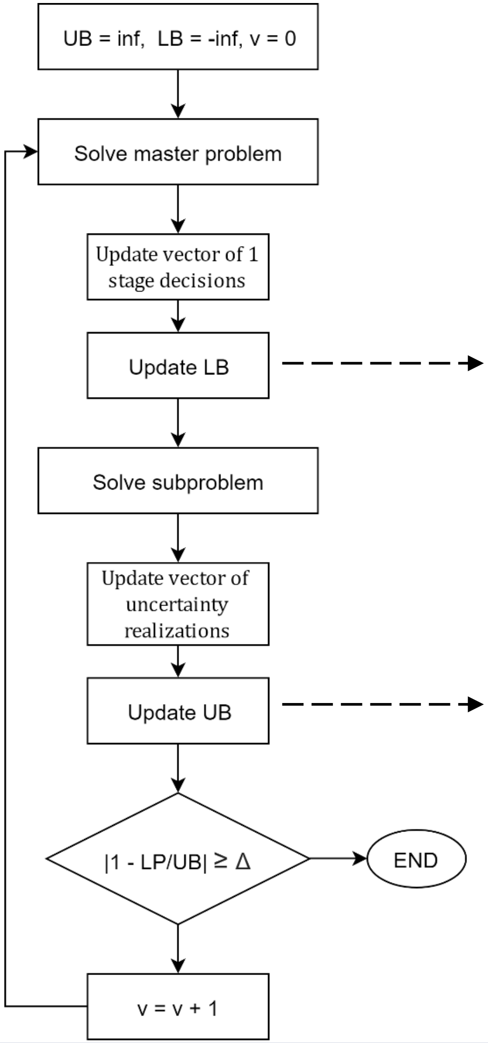
   (they are results of a subproblem)

Subproblem

A **subproblem** is a level 2&3 merged problem [5.aa] - [5.ba] in which a vector of first-stage decisions X_l^{L*} is fixed to a result of the master problem

(this time no equations ☺)

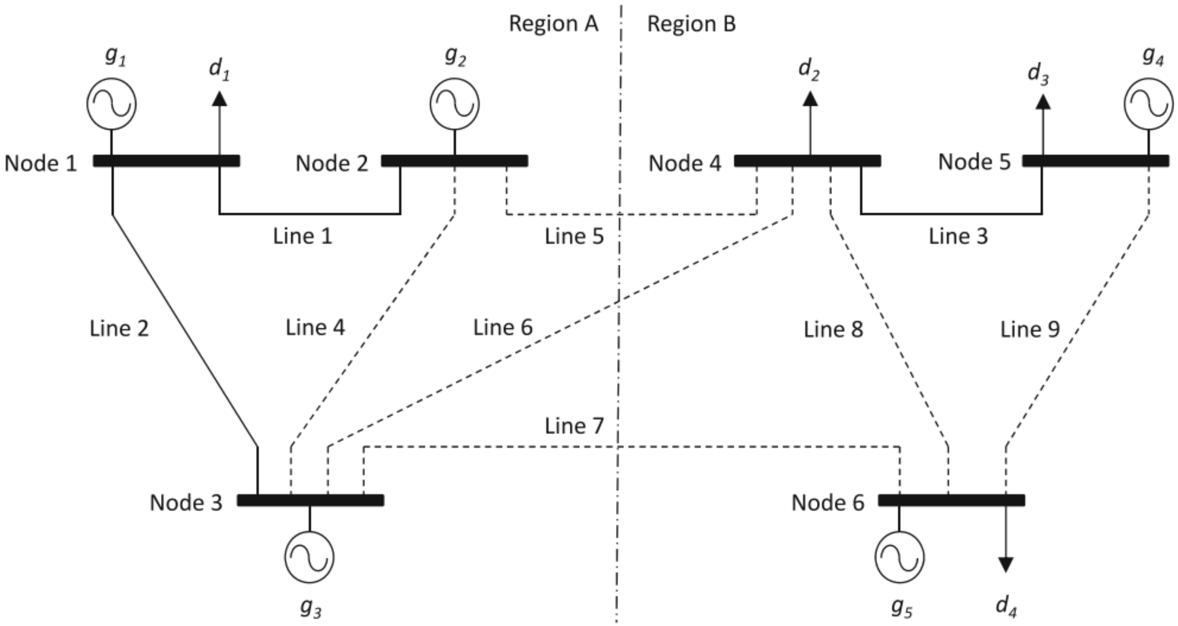
Decomposition procedure



$$LB = \sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^{L*} + \eta^*$$

$$UB = \min\left\{\begin{matrix} UB \\ \displaystyle\sum_{l\in\Omega^{L+}} \tilde{I}_l X_l^{L*} + \sigma\left[\sum_g C_g^E P_g^{E*} + \sum_d C_d^{LS} P_d^{LS*}\right] \end{matrix}\right.$$

Modelling results: sensitivity to uncertainty budget



Line	B_l [S]	F_l^{max} [MW]	$\tilde{I}_l$ [\$] $\cdot 10$^6
4	500	150	0.7
5	500	200	1.4
6	500	200	1.8
7	500	200	1.6
8	500	150	0.8
9	500	150	0.7

Γ^G

Γ^D

	0	0.2	0.4	0.6	0.8
0	l5 l7	l5 l7	l5 l7	l4 l7	l4 l7 l9
0.2	l5 l7	l5 l7	l5 l7	l4 l7	l4 l7 l9
0.4	l5 l7	l5 l7	l5 l7	l4 l7	l4 l7 l9
0.6	l5 l7	l5 l7	l5 l7	l4 l7	l4 l7 l9
0.8	l5 l7	l5 l7	l5 l7	l4 l7	l4 l7 l9

Summing up

Conclusions

1. RO allows for dropping assumptions that a finite number of uncertainty realizations exist with respective (known) probabilities.
2. RO is particularly suitable when decisions are costly and protection against the worst-case scenario is a must.
3. RO allows for robustness control.
4. Accuracy of RO models generally does not depend on the accuracy of the uncertainty description.

Next steps (?)

1. Updating the test case by adding gas system components.
2. Integrating gas system objective/constraints/linkage into the model.
3. Thinking it over…